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Short-baseline parallax in photographs of furnished interiors

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Abstract

A photograph of a breakfast room often contains a patch of apparent sunlight and no visible source. We describe how to estimate the source position from the shadows of equal upright objects on a level surface. In six furnished sets photographed for a housing exhibition, the differential displacement of two shadow tips on a 1.20 m baseline was 90 to 150 mm. The corresponding source heights were 2.25 to 3.58 m above the receiving surface. A control exposure in the exhibition courtyard gave a displacement consistent with zero. Projecting the recovered positions onto the accompanying floor plans put four morning light sources inside neighboring units, one over a common stair and one over an open terrace. Replacing the building by its mirror-image option preserved the recovered height and changed the unit to which the source belonged. The calculation distinguishes distance from direction and angular extent, three properties which are easily confounded in a single photograph.

Keywords: geometrical optics, parallax, shadows, astronomy education, architectural photography

1 A distance in an interior view

On the sales drawing the room faces east. The chair has been pulled back for breakfast and one corner of the table is in sunlight. It is possible to read the time of day from these arrangements in the ordinary descriptive sense. They provide considerably less information about the distance to the source. A light a few meters beyond the window and a light a hundred million kilometers beyond it can cast precisely the same shadow of a single upright at the center of the table.

We were asked to prepare a display about solar distance for a housing exhibition. The exhibition provided six sample rooms, a morning photograph of each and scale drawings showing the rest of the building. We asked for a second exposure of each room with a measured row of objects on the breakfast table. This allowed visitors to recover the distance to the source in the photographs. The quoted distance applied to the morning depicted in the furnishing brochure. An exposure taken outside, at almost the same hour, supplied a second morning with which to compare it.

The exercise follows the small-baseline form of parallax. A nearby object changes direction appreciably between two observing stations. A distant one scarcely moves against its background. A shadow can act as a record of the direction from a station even when a wall conceals the source

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from the camera. The important observation here is a *difference* of shadow lengths. Their common part, which tends to dominate the appearance of the room, can be removed before making any estimate of distance. The calculation also finds the point on the floor plan directly below the source. That point is often the easier of the two results to interpret at an exhibition about apartments.

2 What a row of cups measures

Consider two identical vertical objects standing on a horizontal board. A thin upright with a well-defined top is easier to measure than a drinking vessel with a thick rim. We shall nevertheless call the upright a cup for the moment. If the incoming rays are parallel, translating the cup across the board translates its shadow by exactly the same amount. The vector from the foot of the cup to the end of its shadow remains fixed. A camera can make the two shadows appear unequal through ordinary perspective. Measurements must therefore be made in coordinates of the board rather than in distances along the printed page [6, 16].

Let h be the cup height and H the source height, both measured from the board. Use p for the horizontal coordinate of a foot and X for the horizontal coordinate of the source. Choose positive x going from the window into the room. If the ray through the top lands at q, similar triangles give

$$s = q - p = \frac{h}{H - h}(p - X). \tag{1}$$

The same formula holds separately for the two coordinates in the board. A two-dimensional survey is convenient, but a single row already measures the multiplier $g = h/(H-h)$. For a pair spaced by B along the row the change of the *longitudinal component* of the shadow is gB. A sideways offset of the light adds a common sideways component to both shadows. Taking differences of their total lengths instead would mix the two components. For an obliquely placed source it would usually underestimate g.

Solving for the height gives the working parallax relation,

$$g = \frac{s_2 - s_1}{B}, \qquad H = h\left(1 + \frac{1}{g}\right). \tag{2}$$

For a nearby source the upright further from the window casts the longer shadow. If the rays are parallel the numerator in Eq. (2) vanishes. In that case the appropriate result of the experiment is a limit on g. Dividing by a noisy number close to zero can produce a large negative height as readily as a large positive one. It is better to leave a courtyard result on the scale on which one read the ruler.

There is a direct order-of-magnitude check. With $h = 0.25$ m, a 1 m baseline and a source 2.75 m above the board, Eq. (2) gives a differential displacement of 0.10 m. For a source overhead at one astronomical unit the displacement would be about 1.7×10^{-12} m. At lower elevations the vertical height of the Sun is smaller by the sine of its elevation; at thirty degrees this merely doubles the result. We use the conventional astronomical unit [8]. The room need not be accurately oriented or the observing date known to distinguish ten centimeters from a few picometers.

3 The boards and their placement

We examined all six sets for which the exhibition had commissioned a view described on the picture list as morning. The rooms occupied an open stage at the edge of the exhibition hall. Only the walls and fittings seen by the camera had been built. The accompanying plans included the other apartments on each floor, with dimensions and room uses. These drawings made it possible to continue the coordinate system of a photographed table into space absent from the stage. No one lived in the adjacent apartments. Their room uses below refer to the architectural schedule supplied with the display.

For each reference exposure the photographer placed four 250 mm uprights in the holes of a folding board, their centers 400 mm apart in a straight line. Each had a small opaque horizontal blade at its top. Its central engraved notch served to identify the shadow of the blade center. A dark collar marked the foot. The uprights replaced a row of cups on the table for the reference frame. Cup handles had proved troublesome in a rehearsal. Depending on rotation, a handle could cast a separate shadow beyond the end of the shadow of the rim [2]. The camera stayed on its head while the board went in. After the reference frame the ordinary furnishing exposure was made with the board removed. The electrician’s settings, window dressing and camera position remained fixed for these two exposures.

The camera saw most boards at a steep slant. We put eight marks around the board rim and surveyed them with a steel rule. Four widely separated marks determine the plane-to-image transformation; the other four check it [6, 18]. We corrected radial lens distortion using a grid exposure at the same focus setting before solving for the transformation. A check mark left out of the solution lay within 1.7 mm of its surveyed position in each room. The transformation applies to objects on the receiving plane. Applying it to the visible top of a blade would place the top at the wrong horizontal coordinate. We used it only on the collar centers and the tips of the *shadows*.

A white tablecloth would have shown an edge but not a ruler. The board instead carried a matte gray sheet ruled in dots. We measured the midpoint of the intensity change along each side of the notch and averaged the positions. A soft shadow retained two readable inflections. A shadow with folds or distinct secondary edges failed the reading rule [11]. We could move the whole board laterally on the table before recording the frame. In set II the first orientation put one notch across the shade cast by the window frame. We rotated the board within its plane and used the angle recorded on the placement card to resolve the measured vectors along its new axis. This did not require moving the window frame or assigning an arbitrary endpoint to a compound shadow.

In each of the six sets the technician removed and replaced the board four times. All 24 accepted frames contained four readable shadows. A separate exposure of the board leaning against the table was retained for inspecting the notches but made no contribution to a length measurement. We did not regard the four uprights as four independent repetitions of the lighting setup. Their positions together determine one slope. The variation between placements includes centering and leveling of the board. Changes in source position during the morning would be confounded with placement drift. The entire series in each room took less than eight minutes, with the stage lighting controls locked.

4 Recovered heights

The variation with baseline in room III is shown in Figure 1. Each plotted value uses the first upright as one endpoint and the named upright as the other. The three nonzero points share the first reading, so treating them as independent would understate the uncertainty of a line fit. For the room survey we fitted the four individual shadow offsets as a function of the four foot positions, allowing a free intercept. We did this separately on each of the four frames and averaged the slopes. The plotted error bars describe the scatter between placements; the value at the origin is identical to itself.

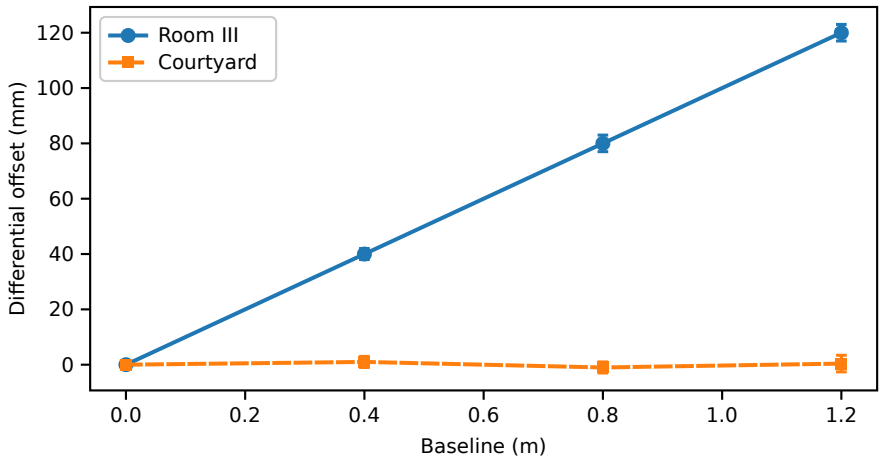


Figure 1: Change of longitudinal shadow offset from the first upright. Bars show one placement standard deviation in room III and in the courtyard.

For ease of checking by eye, Table 1 gives the equivalent difference across the full baseline, calculated from the fitted g and rounded to the nearest millimeter. The height above the board is obtained directly from Eq. (2). The room with the longest difference is the one whose source has the lowest height. Short shadows in that room need not accompany a low source; shifting the source horizontally toward the board shortens every shadow without changing their differences.

Table 1: Per-room parallax solution. The equivalent tip difference equals 1200 mm times the fitted slope. The final column is the floor-plan region beneath the source center.

Set	Slope	Difference (mm)	Height (m)	Region
I	0.075	90	3.58	Unit 2, bedroom
II	0.090	108	3.03	Unit 4, linen closet
III	0.100	120	2.75	Unit 3, bedroom
IV	0.110	132	2.52	Common stair
V	0.125	150	2.25	Unit 6, study
VI	0.080	96	3.38	Open terrace

Uncertainty in blade height was 0.5 mm, taken as a standard uncertainty for the common measured height. For set III the placement mean had a slope uncertainty of 0.0014. A change of

0.5 mm in blade height changes H by 5.5 mm at $g = 0.100$. Treating the measured length scale, readout and placement errors as separate at this precision gives the budget in Table 2. Terms shared by all shadows cannot be reduced by observing more uprights on the same board [7, 19]. We use an estimated standard uncertainty of 0.05 m for the height in III. It describes the geometrical fit for the photographed setup. The spread across rooms exceeds this by more than an order of magnitude.

Table 2: Standard uncertainties in the height above the board in set III. Scale and registration terms were varied once per complete frame.

Contribution	Variation used	Effect on height (m)
Placement mean	0.0014 in slope	0.035
Residual plane mapping	0.0009 in slope	0.023
Baseline length	1 mm in 1200 mm	0.002
Common blade height	0.5 mm in 250 mm	0.006
Shadow edge convention	0.0010 in slope	0.025
Quadrature total		0.049

In the courtyard the staff used the same table and board in direct, unobstructed sunlight. The fitted slope was 0.0003 with a standard uncertainty of 0.0020. A two-standard-uncertainty upper value of 0.0043 would put the source above about 58 m relative to the board. This limit is a property of the measuring equipment. It is quite compatible with the usual astronomical value, and also with many considerably less remote objects. The height inferred by simply inserting 0.0003 in the denominator would have no useful precision. An outdoor control thus distinguishes the local and distant regimes without purporting to measure the Earth-Sun distance from a folding table.

5 A plan position from the common shadow

The intercept discarded in taking the difference contains the other half of the result. For each blade, Eq. (1) gives

$$(X, Y) = (p_x, p_y) - \frac{(s_x, s_y)}{g}. \tag{3}$$

Four blades give four estimates of the same point and can reveal a failure of the single-source model. We averaged the four reconstructed points within each placement, keeping one common fitted slope. Black alignment tabs on the underside of the board set its position relative to the table. The table corners were in turn surveyed relative to the inside face of the window wall and its left jamb. Those two physical marks appear on the supplied plans. Their separation supplies a check on the stated scale without the need to identify a piece of movable furniture in a sales drawing [10, 13].

Set III supplies a numerical example. Its first foot was at $(1.00, 0.40)$ m relative to the window and jamb. The longitudinal shadow extended 0.400 m away from the window and its signed transverse component was minus 0.040 m. With the slope 0.100, Eq. (3) places the source at $(-3.00, 0.80)$ m. The last foot lay at $(2.20, 0.40)$ m and had a longitudinal offset of 0.520 m.

Both feet therefore put the source the same distance beyond the wall. Relative to the first foot its slant distance was about 4.87 m. The number 2.75 m in Table 1 is the height above the board; it should not be used as the radius of a sphere centered on the first foot.

Projecting the center into the reference floor puts it 3.00 m beyond the window wall, in a bedroom assigned to unit 3. All references to bedrooms in Table 1 are horizontal projections. Adding the table height to the source height may put the light above the finished ceiling of that room. Each plan belongs to its own design. Unit numbers from different rows do not describe residences in a single building. In II the center projected inside the linen closet in unit 4. Its nearest drawn partition was 0.31 m away. Moving the window registration to either edge of its printed line changed the distance to that partition by 0.04 m.

To check whether a room assignment depended on a fine distinction at a boundary, we measured the signed clearance from the plan point to its nearest change of use. A common stair borders two units, for example, and crossing its edge would transfer a source from a common area to one unit. The distances and propagated measurement uncertainties appear in Figure 2. For each room we displaced the fitted slope and intercept together according to the covariance of the line fit, then added placement and wall-registration uncertainty. The bars cover approximately two standard uncertainties in the distance normal to the nearest boundary. They exclude errors in the architectural plans themselves. Even for the linen closet the indicated interval lies inside the same room.

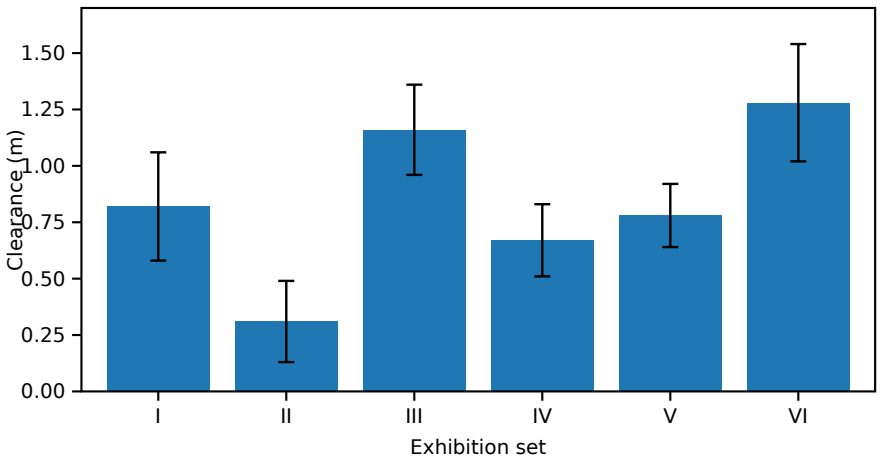


Figure 2: Horizontal distance of the projected source center from the nearest floor-plan boundary. Bars are twice the estimated standard uncertainty normal to the boundary.

We checked a plan variant offered for set III. Its illustration reversed the picture left to right and put the breakfast room in the mirrored end unit. A plan with all room numbers, doors and wall thicknesses was available for each hand. A reflection preserves the measured shadow divergence and hence the source height. Transferring the coordinates with the reflection put the projection in the corresponding bedroom of unit 5. Using the original photograph with the reversed plan without changing the sign of its transverse coordinate placed it in the passage. The two assignments are distinguishable by locating the window latch. From this small detail it was possible to establish which unit supplied the daylight for the reversed listing.

6 Size and distance need different measurements

A second impression of distance comes from the hardness of the shadow. A small distant disk and a small nearby disk can subtend the same angle. Hardness alone will not separate them [12, 14]. For the nominal solar radius and one astronomical unit [8, 9] the solar diameter subtends about 0.53 degrees. A source only 5 m away could have that diameter in angle if it were about 46 mm wide. To reproduce its shadow across multiple cups it would still have to pass the parallax test. Reducing the size of a local source narrows the penumbra without removing the divergence of its central rays.

The transverse component of the experiment offers a simple finite-source calculation. Put two equally bright point sources at the same height and displace them sideways by a distance d. By Eq. (1), at a given blade their shadow tips are a distance gd apart. A uniform luminous strip filling the interval smears the edge across that same width. One can use the 10 to 90 percent width of the change in intensity to suppress the dark tails of a noisy profile. For a straight knife edge illuminated by a uniform strip this width is $0.8gd$. This coefficient changes for a circular disk or a graded diffuser [15, 21]. We used a vertical knife-edge gauge with a horizontal upper blade at the same height as the original uprights; the scanned edge ran transverse to the direction along which the source width was measured.

For set III the transverse 10 to 90 percent width was 8.0 mm after allowing for the camera response. Under the strip model that implies an emitter 0.100 m wide. At the measured slant distance the corresponding small angle is about 1.2 degrees for a width transverse to the line of sight. The reported width is an equivalent transverse width. A reflector high on one side of a lamp could make it wider without moving the filament. The calculation has the practical merit of expressing the extent of the source on the same plan on which one has already placed its center.

Independent checks of that calculation used removable masks over a flat rectangular test light at the reconstructed height. The mask widths were measured at the luminous plane. We obtained the curve in Figure 3 by measuring the 10 to 90 percent widths with three masks. The straight line is fixed by the measured slope of set III. There was no adjustment of its slope to the width data. The broadened transition in the furnished frame is marked at the equivalent width for comparison; that single datum is an inversion and supplies no independent evidence for the strip law. Scale and mask calibration followed the contact-screen procedure of Devainel [3].

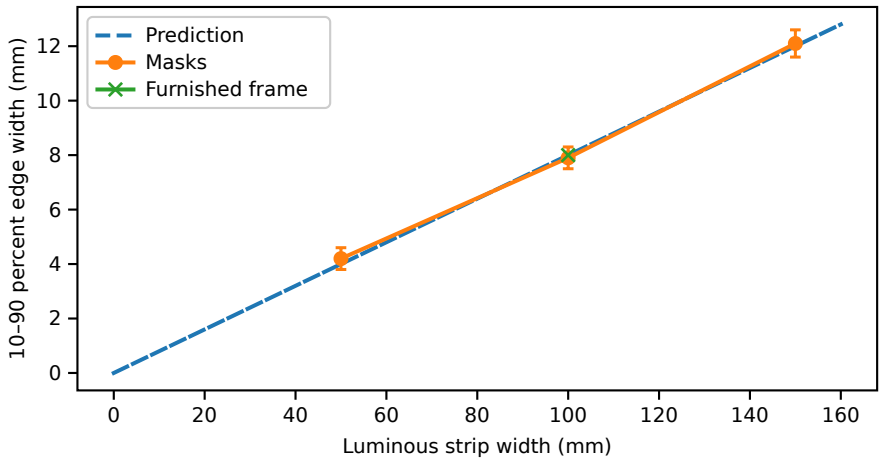


Figure 3: Transverse edge width for three measured luminous strips at the height of the set III source. The line is calculated from the parallax slope.

7 Reading a public photograph

The reference board supplies information which the printed room photograph seldom carries on its own. To recover a metric height from a published photograph, one must know a length, identify the receiving plane and match at least two shadows to objects of known height. A set of supposedly identical wine glasses is a less reliable scale than it looks. The near glass can stand on a tray and a rear glass can be partly filled, with a bright caustic crossing the apparent shadow of its foot [1]. For two glasses of unequal height, imposing a common factor in Eq. (1) absorbs the height difference into a spurious parallax slope. Straightening the camera image will do nothing to remove this error.

There is a broader ambiguity about the word source. A sunlit window blind can illuminate a room by reflection. A point reconstructed near it may locate an effective luminous region of the blind even when the ultimate energy comes from the Sun. A projection optic can send nearly parallel rays from a lamp just outside the window. That arrangement can pass a distant-light test on the board while the housing around the bulb is quite close [4, 17]. In optically complicated rooms the result concerns the bundle of rays that makes the measured shadow. It need not locate a manufactured emitter. We expressed the results in terms of a source for that reason, and did not use the location of a cable or its power outlet as a distance check.

Visitors could place both a fingertip and a photograph at once on the scale drawing. Several were able to locate the projection more easily by extending the shadows on tracing paper. That method fails for parallel shadows and becomes sensitive to small reading errors at great distance. In the linen closet the intersection was clear. For the equivalent outdoor exposure it lay off the sheet even when the drawing was extended to include the adjoining building. The value of retaining the short-baseline measurement in that case is precisely that it still has an answer, in the form of a bound on the divergence.

A Reduction of a worked exposure

With a horizontal blade at fixed height in set III the four longitudinal foot positions were 1.00, 1.40, 1.80 and 2.20 m from the window plane. In an individual reference exposure the corresponding offsets read 399, 441, 479 and 521 mm. Subtraction of the first from the last gives 122 mm across 1.20 m, or 0.1017. Fitting all four positions yields 0.101 to three decimal places. Averaging the four placement slopes gives the rounded 0.100 in Table 1. Rounding the positions to the nearest centimeter before subtraction would overstate the instrumental precision of the calculation. In particular it would conceal the small differences one is trying to resolve in the courtyard.

The two-dimensional residual offers a check which a visual intersection lacks. Having fitted g to the longitudinal data, subtract g times each foot's coordinates from its measured offset in both axes. The residual vectors should agree across the four positions. Differences transverse to the row can flag a tilted blade, a misread edge or a second direct light contributing appreciably to the chosen edge. Differences common to all four points shift the fitted plan position. They have little effect on the fitted height. A registration check done only by comparing heights would miss a uniform 10 mm offset in shadow readings, which moves the projection by 100 mm when $g = 0.100$ [20].

The 0.0014 placement term in Table 2 is a standard error of a mean over four placements, obtained from a sample standard deviation of 0.0028. It does not include changes made when switching lighting directors or moving equipment to a different hall. The edge term is a sensitivity allowance: we changed the edge threshold on all four uprights at once and examined the change

in the fitted slope. Response linearization used the gray scale imaged beside the board. Applying a threshold directly to display brightness can move a nominal midpoint toward the dark side, particularly in brochure images with adjusted contrast [5]. The housing exhibition allowed the use of reference exposures before that adjustment. The printed plans were adequate for deciding which apartment contained the reconstructed projection, but their line widths were already a significant part of the uncertainty for a closet door.

Acknowledgments

The exhibition workshop kept the reference frames with their placement cards and supplied the floor plans for both hands of the end unit.

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