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Transmission of an obsolete boundary through successive editions of a folded map

Miren U. Ardanet,[1, *] Petar I. Belovac,[2] and Aoi N. Tsunemori[1, 3]

[1]*Laboratory of Sheet Mechanics, Arven Technical Institute, Lausanne, Switzerland*
[2]*Department of Cartographic Materials, Drenica College of Surveying, Rijeka, Croatia*
[3]*Division of Fiber Physics, Kitahama School of Applied Science, Kobe, Japan*

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A fold introduced at a territorial division may outlive the ink that first marked the division. We ask whether it can also outlive the paper. We replaced a large-scale plan six times by folding each new edition around its predecessor and then withdrawing the predecessor. The boundary had been abolished before the first replacement. All subsequent editions show the same undivided territory. With an imposed displacement of 2 mm between the stock fold and the former frontier, the fifth replacement still opened at the frontier in 31 of 48 independent chains. A flat transfer gave 4 of 48. Cross sections show a narrow zone of residual curvature moving toward the stock fold at each exchange. Transfer ceases when that zone is removed from the incoming sheet or when the old sheet is withdrawn before the first closure. An enclave survives along its long sides after its two short sides have disappeared. We give the range of registration errors over which a physical copy of the former division can be distinguished from the stock crease.

I. INTRODUCTION

The boundary in one series of valley maps passes through the center of a millpond. There is no feature there to sight on. Nevertheless a sheet laid on a light table rises slightly in the middle of the pond, and on closing the sheet the two former jurisdictions fall on opposite faces. On the first edition this is easily explained. The printer supplied separate plates for the two districts and used the division as a delivery fold. The puzzling sheets are replacements made after the districts were combined. The millpond is now printed in one piece. Some of these sheets will still stand on the old frontier with both banks clear of the table [1, 2].

The usual account of inherited folding assumes an operator who knows how the predecessor was folded. That assumption is difficult to test against surviving maps. Copies go into identical cases, and a sleeve seam or the edge of a board can make the same line on sheets separated by a century [3]. We therefore reproduced the change of edition on a press in which no choice of fold is available to the operator. A closed map serves as the mandrel for the next edition. Once the fold has been made there is no continuity of matter between the two editions. This permits an experiment with a fairly strict definition of succession. We count a jurisdiction as mechanically present if opening the sheet separates it from its former neighbor along a line within a specified distance of the original frontier.

In a single sheet, an old fold has a preferred dihedral angle and competes with bending of the adjoining panels [4, 5]. Repetition can preserve a history of earlier forcing even when the mean shape is featureless [6]. In the map problem the loss of the carrier is essential. Measuring the same crease over six closures would test relaxation. Here the instrument interrogates a strip whose fibers have never been in contact with the sheet on which the boundary was printed, except at the first replacement. Neither a mark transferred in ink nor a photographic record of the old edition is available to the reader. The question is how much of the displacement survives the finite width of a paper hinge at each exchange.

II. SPECIMENS AND CHANGE OF EDITION

Our master has two rectangular districts sharing a straight boundary on a 240 by 180 mm sheet. The nominal delivery fold on the new drawing lies 2.00 mm east of the former boundary. The backing board locates the eastern and western edges in slots; it has no score line. We use west as the positive direction when giving displacements from the delivery fold. Each chain starts with a cut from a fresh parent sheet and a single line impressed over a rounded brass lip on the western coordinate. We print the frontier on its inner face for identification before the chain begins. Every replacement comes from the undivided edition, on which the ink coverage in the observation window is uniform. Two edge fiducials outside the window set the longitude scale. They are cut off before the opening test.

We trimmed sheets after a day in the conditioning cabinet. Stock S was an uncoated wood-free book paper of thickness 94 ± 3 micrometers. Stock T, used for the width study only, was 132 ± 4 micrometers. The uncertainties are standard deviations of fifteen parent-sheet means taken at five positions on each sheet. Running the fold parallel to the machine direction mattered more than the variation in thickness within S. The tensile stiffnesses per unit width in the machine and cross directions were 418 and 214 kN/m. The bending stiffnesses measured on cantilever strips with the curvature normal in those directions were 0.32 and 0.17 mN m.

We use the latter number for a longitudinal fold. The difference between a tensile and a flexural estimate is ordinary in a material with dense outer layers [7, 8]. We made no conversion using an assumed Poisson ratio.

During exchange the two leaves of the old edition sit between the two leaves of the new one. A soft roller closes the packet from its open edge toward its spine over a free roller bed. The end stops give a small bending moment toward the stock coordinate as the fold closes. These stops hold only the long panel ends. Slots within a millimeter of the old spine would constitute a second forming die. The packet spends 30 s under 0.75 kN/m line load at the final closure, then opens through 50 degrees. A traveling jaw draws the old sheet out along the spine. We keep the new sheet closed until the next roller cycle. One edition is one passage of fresh paper through this sequence, including withdrawal. No folded wrapper or interleaf passes through two exchanges. A second technician counted the discarded editions at the end of each chain.

Registration was by the cut edges, using the same stops whether a previous edition was present or absent. The flat-transfer arm used the old edition opened to 180 degrees and removed it before closure. We built the same thickness into the end stops in this arm. An additional series used a virgin sheet folded around an old map supported on its flat face with the spine projecting beyond the roller. This last operation retained contact, ink, and sliding during withdrawal but removed contact at the line in question [9]. All sheets had a 45 mm inspection margin at either end of the fold. The center 90 mm supplied three adjacent 30 mm strips after the assigned number of exchanges. Cutting is destructive; a chain used for edition two appears in no later count.

The collection was divided into eight runs. Each run supplied six independent chains for every closure condition and stopping edition in Fig. 1. Parent stock was apportioned among conditions within a run. A separate set of 24 chains at each of the five initial offsets supplied profiles for Fig. 2. The reader received random strip numbers and the location of the new stock fold. Neither the direction nor the size of the imposed offset appeared in the strip number. Mirror-image trials used the same cuts rotated end for end in the plane of the paper. Their coordinates were reversed during analysis. If a machine bias survives that operation it appears with opposite signs in the two halves of a run [10].

III. OPENING COORDINATE AND TRANSFER LAW

We impose a displacement on the ends of a strip in the plane normal to the spine until the two far panels make an angle of 130 degrees. A telecentric camera records the edge in silhouette. The local tangent angle is obtained by a cubic fit over 0.6 mm, and the position of maximum absolute curvature gives the opening coordinate. Where two peaks differ in height by less than 5% we report both and count the strip as unresolved. Strip readings from the same sheet serve as subsamples. We took the median position among the resolved strips as the chain reading. Fewer than two resolved strips left the chain unresolved. Unresolved chains stay in the denominator. Blind duplicate tracings differed by 0.09 mm root mean square, appreciably less than the separation needed to classify the sheet [11, 12].

Our threshold for the boundary is a band extending 0.80 mm to either side of its original coordinate. An instrument tracing placed at precisely the stock coordinate consequently falls 1.20 mm short of the nearest acceptable line. The width is that of the narrowest base line on the master at map scale, enlarged to include one full pixel of uncertainty on either side. It was specified when the master was ordered. For offset experiments closer than the band width we report only peak positions; a binary identification would then count the stock line with the frontier. On surviving maps one must know the registration between the original plate and the edges to make either measurement. A drawing cropped in the middle of the millpond carries insufficient information.

Consider a closed predecessor carrying its spine at x. Its outer jacket has a compressed strip of width w about x. Over a small interval near the two competing positions we approximate the work to form the new hinge at u by

$$W(u \mid x) = \tfrac{1}{2} K_s u^2 + \tfrac{1}{2} K_c (u - x)^2 + C. \tag{1}$$

The first term in Eq. (1) is the work against the registration stops. The apparent contact stiffness K_c includes bending over the predecessor, pressure at the edge, and whatever yielding is needed to hold a trace after withdrawal. Both coefficients have units of energy per squared travel for a specified strip length. The expression follows a local expansion of the measured work about each preferred position; nothing in it requires the sheet to be a linearly elastic solid during closure. In particular the intercept C can include a large irreversible loss. Energy measured on opening the strip cannot be substituted for the work of forming it [13, 14].

Minimizing Eq. (1) gives a fractional transmission $a = K_c / (K_s + K_c)$. If the next sheet has an edge registration error e_j, independent of the sign of the offset in the previous sheet, the coordinate after j exchanges obeys

$$x_j = a x_{j-1} + e_j, \qquad E[x_j \mid x_0] = a^j x_0. \tag{2}$$

This law is local in x. A fold four or five hinge widths from the stock crease produces two separate ridges and in that regime there is no reason to expect a single minimum. We measured the first derivative of Eq. (2) by paired offsets at the first exchange. Slopes were fitted through an estimated intercept; forcing a zero intercept discards a direct check on the edge stops. On S the first-exchange slope on offsets from minus 2 to plus 2 mm was 0.93 (run-resampling interval 0.90 to 0.96). The intercept was 0.03 mm (−0.05 to 0.11). A 2 mm initial displacement would place the fifth opening near $2(0.93)^5 = 1.39$ mm, about 0.19 mm inside our acceptance region. Drift of a single ruler graduation could therefore affect a substantial fraction of the retention counts.

IV. RESULTS

At the first exchange, raising the undivided edition from the bed almost always opened it along the former frontier. We could hold the folded sheet up by the district that no longer had a separate map plate. Counts at later exchanges are in Table I. The project statistic was the fifth-exchange difference between the two arms, specified before the strips were decoded. Its estimate was 27/48, or 56.3 percentage points. The eight run differences, in sheets out of six, were 2, 3, 3, 4, 3, 3, 5, and 4. Resampling the eight complete runs as matched units gave a 95% percentile interval from 45.8 to 66.7 percentage points. The interval pertains to stock S at this press setting. The paper lot and the roller bed are fixed, and repeating a run does not sample new lots or presses [15, 16].

TABLE I. Opening within 0.80 mm of the old boundary. Each denominator is 48 separate chains; unresolved sheets are included as failures.

Exchanges	Nested	Flat transfer	Unresolved, nested
1	47	3	0
2	44	4	1
3	41	2	1
4	36	3	2
5	31	4	3
6	25	3	3

The continuous readout helps distinguish decay of the fold from a discontinuous redistricting of individual sheets. The upper quartile and median move toward the stock coordinate together. Among the 45 nested sheets with a unique answer at exchange five the median was 1.40 mm and the quartiles were 1.07 and 1.63 mm. At that edition the width of the central interval exceeds its distance from the cutoff. Readers applying the binary test in Table I would reasonably disagree about a sheet held only by its edges. The sign of the displacement, on the other hand, was correct for 43 of those 45 sheets. We do not include sign agreement among the successes because a displacement of 0.1 mm in the right direction is too small to restore a district on the stipulated scale.

The upper and lower sequences in Fig. 1 were read at the same angle. The flat controls occasionally opened around a random soft spot in the band. These few openings were dispersed along the press rather than concentrated at one position on the width of the roll. Blind relabeling of the west and east halves of the flat-control set left their count near its original value. The projecting-spine control produced three positive sheets in 48 separate five-exchange chains. Retaining contact with the face of the predecessor appears insufficient when the ridge itself does not pass under the roller.

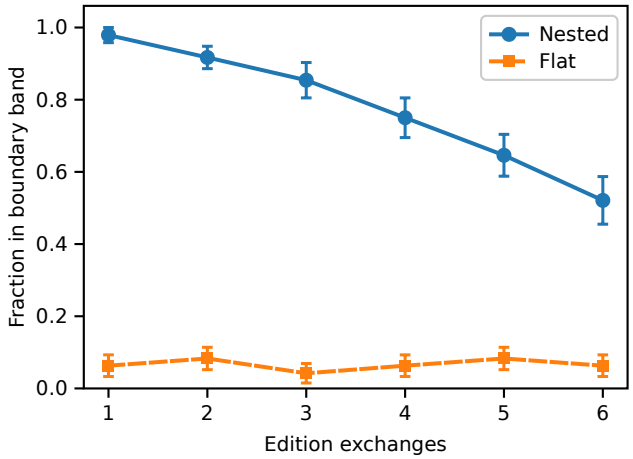


FIG. 1. Fraction opening at the old frontier, with eight-run standard errors. Each point uses a separate group of 48 chains.

At first transfer the two grades give different slopes (Fig. 2). The outer arc on the thicker grade occupies a larger fraction of the distance to the stock coordinate; its contact patch also broadens under the roller. A wider patch can reduce the preference for a particular line. Substituting the coefficients into Eq. (2) gives K_c/K_s of about 13 for S and 4 for T. This is an effective ratio under the imposed closure; it does not identify a separate friction coefficient. Holding the predecessor flat under a line load gives a very different contact patch. Nor can these data resolve a change of contact stiffness from a change of bending work in the panels. Both change with thickness [17, 18].

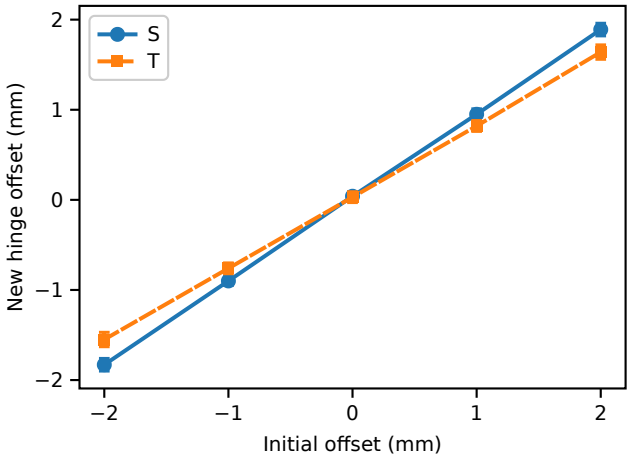


FIG. 2. First replacement. Mean coordinates from 24 chains per offset and grade. Bars give standard errors across runs.

A. An enclosed district

The common frontier had no ends in the inspection window. A closed enclave supplies four ends and forces compatibility at the corners. We tested its transfer on a separate 160 by 100 mm district drawn around a 70 by 24 mm rectangle. A box-fold jig put the two short edges and then the two long edges onto the master, producing four residual ridges. We withdrew the master sideways with the long leaves half open. New editions bore only one uniform district. To keep the scoring of adjoining edges independent of corner kinks we omitted 5 mm on either side of each corner. A closed enclave was recorded only if all four free edges fell within their respective bands. A good long-side reading can thus be accompanied by failure of enclosure on the same sheet [19].

In 32 new nested chains for each stopping edition, the counts of mechanically closed enclaves at editions one, three, and five were 25, 10, and 2. Corresponding counts retaining both long sides were 30, 25, and 21. Among the 19 fifth-edition sheets carrying two long sides and an incomplete end, the visible region runs off the end of the former district; it no longer encloses a particular area. Counting surviving millimeters of frontier alone would assign those sheets substantial territorial persistence. We used a topological requirement in the record form because the specimen was ordered as an enclave. Even a 1 mm gap permits a pencil to be drawn from its interior to the surrounding district without crossing a surviving fold.

The edge-by-edge readings in Fig. 3 show a directional effect. Sides parallel to the machine direction survive for more exchanges. Their advantage includes the original order of closing the jig, which always made the long folds last. Rotating the die by a quarter turn while holding machine direction fixed would separate the two contributions. It would also move the withdrawal jaw, which was too short to clear the rotated rectangle on this run. We consequently limit the reading to survival of enclosure in this geometry. The two residual long edges give no independent estimate of a material anisotropy.

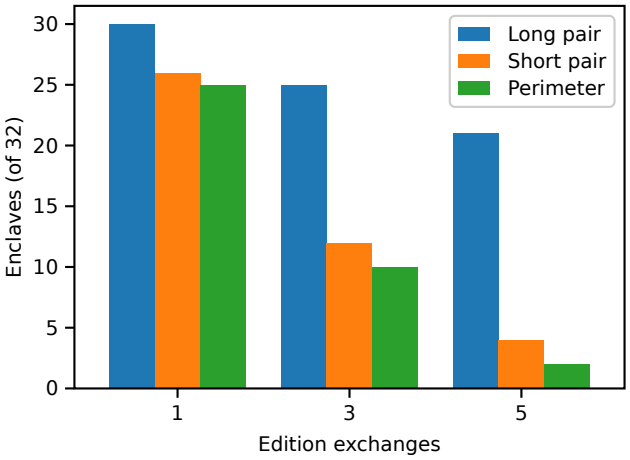


FIG. 3. Numbers of enclaves, out of 32 at each stopping edition, retaining the specified parts. The complete perimeter is a subset of each two-side count.

V. SENSITIVITY AND PHYSICAL SCALE

An estimate of the instrument’s opening work can be made without access to the roller. Bending a strip of active length $l = 30$ mm through an extra angle $\theta = 0.5$ rad over an arc of width $w = 2$ mm costs about $D l \theta^2 / (2w)$ or 0.32 mJ on S. The opening fixture delivered 0.28 to 0.46 mJ over that increment on five sacrificial strips. The correspondence in scale is consistent with a hinge that shares its deformation with a few millimeters of neighboring panel. Replacing w by the 0.1 mm sheet thickness would overestimate the work twentyfold. Measuring only the radius of the tightly closed ridge on the master cannot supply the hinge width used in an opening calculation [20].

Uncertainty in a signed opening coordinate has two rather different sources. A camera error moves the estimated position of one curvature peak. A change in humidity or in the path of the jaw can cause the competing peak to become the larger of the two. We summarize the former contributions in Table II and report peak switches separately. Repositioning the camera between paired photographs left the opening unchanged. Releasing and regripping the specimen, in contrast, changed the largest peak on 3 of 40 duplicate strips. Adding those events as a Gaussian camera error would distribute a rare shift of about 1 mm across every photograph. The quoted distributions use the first grip

throughout. Duplicate readings entered only the sensitivity calculation [21, 22].

TABLE II. One-standard-deviation uncertainties for the coordinate of a single resolved peak on S. The shared datum term is common to one run.

Source	mm	Basis
Edge fit	0.06	Duplicate silhouettes
Pixel scale	0.02	Graticule
Datum setting	0.05	Independent reset
Strip placement	0.08	Repeated seating
Quadrature	0.11	Independent components

There is a separate uncertainty about what vanished when the boundary was repealed. A residual sheet left loose inside a case can survive all six changes of edition. If the roller continues to fold around it, each new sheet is a direct descendant of the master. Our procedure counted discarded sheets and opened the nest between passages specifically to rule out that history. It would be hard to impose the same condition in an archive. A dated frontier with fresh paper on both sides might represent one recent transfer from an old plate, several generations of mechanical copying, or reuse of a sleeve. The fitted loss per generation applies only to the second case. It cannot supply a date without the genealogy of the carrier [23].

For conservation work this leaves a useful distinction at the millpond. A line of ink records a plate. A band of high curvature records a load under particular registration conditions. A stock fold that misses the blank stretch of the pond by two millimeters can still open in the middle of it after several changes of stock. It follows that an inventory made by opening each folded sheet and recording the districts on its free faces can contain one more jurisdiction than an inventory made by reading the same sheets flat.

APPENDIX A: DERIVATION AND LIMITS OF THE COORDINATE ESTIMATE

The sign convention used in Sec. III places the first frontier at positive x_0 after reflection. Iterating Eq. (2) gives an error contribution $e_j + a e_{j-1} + \cdots + a^{j-1} e_1$. For independent zero-mean errors of variance s^2 the conditional variance at exchange j is $s^2 (1 - a^{2j})/(1 - a^2)$. If all error originates in the datum of a single run, each term has the same sign and the factor becomes $[(1 - a^j)/(1 - a)]^2$. The distinction matters by the fifth exchange even when the single-pass repeatability is good. We measured the scale graticule at the beginning and end of each run, keeping the two readings together under resampling. Dividing a run effect by the square root of the number of strips would assign the precision of many repeated sightings to a single setting of the datum.

Three exclusions bear on extrapolation. First, a wet sheet can take a sharply localized set at a much lower load. All undivided editions in this experiment went from cabinet to bed within two minutes and returned to the cabinet between exchanges. Second, a polished mandrel made of two metal leaves held the offset to within 0.1 mm in six trial exchanges, but in that case we used the original mandrel throughout. It establishes the upper limit of registration by the bed. Finally, our replacement procedure sheds a sheet even when its fold is excellent. A case worker may keep an unusually well-creased master for years. A survival distribution for issued maps would have to sample this retention preference as well as the transfer process measured here.

The edge test uses curvature magnitude, which is unchanged if the reader opens a mirrored copy from the wrong side. A historian seeking evidence of eastward annexation would need the sign relative to the orientation of the sheet as issued, supplied in our tests by the clipped coordinates on the press log. Once both edge fiducials and the ink have been lost, the surviving hinge can partition the sheet. It cannot say which district acquired the millpond.

ACKNOWLEDGMENTS

The Arven print room cut the two undivided editions from a common drawing and retained the register keys until the strip readings were complete.

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