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Pauli-frame recovery with conditional delivery of classical messages

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Classical delivery rules can suppress or separate components of a Bell-measurement record even when the record is available at the source. We characterize the resulting Pauli frame in a trapped-ion teleportation experiment with optical readout and deferred classical correction. A receiver restricted to a return-date field retains the population transfer and loses both transverse components. Full delivery gives a six-state fidelity of 0.9467 (95% interval $[0.9424, 0.9509]$). With one response permitted per sender, fidelity is 0.6114 across all attempts and 0.9467 among delivered attempts. We recover 0.9442 by reconciling two independently routed copies of a printed automated absence notice. The primary copy supplies the date of return, while the secondary supplies the name of the interim contact. Both copies are required to recover the three components of the Pauli transfer.

I. INTRODUCTION

Teleportation separates the transfer of an unknown state into an entanglement resource and a classical message [1]. The message is short. For the usual qubit circuit, it takes one bit to identify the required X correction and one bit to identify the required Z correction. Experimental accounts quantify the fidelity of the entangled pair, the Bell measurement and the receiving qubit in some detail [2, 3]. The electronics conveying the two classical bits are generally specified by their latency. In laboratory time, a correction arriving after readout is late. It can still determine the sign of a Pauli measurement whose basis was fixed before it arrived. Tracking these signs is routine in measurement-based protocols [4, 5].

Suppressing a whole record gives an erasure at a known location if the receiver has a manifest of expected arrivals [6]. The arrival of an identified record leaves open whether the service delivered both bits. This distinction occurs in forms that disclose an internal coverage assignment to a different group of recipients from the return date.

We send the two correction bits using the return date and interim contact on a printed automated absence notice. Under the records office's default rule, only the first request from each address receives a reply. The receiver detects the missing trials from the manifest.

The experiment uses a complete local Bell measurement, as in deterministic ion demonstrations [7, 8]. Alice and Bob occupy separate zones at the same station. For retrospective verification we apply the frame to recorded Pauli readout. A live output would require a receiving memory and a delivery deadline, as in remote matter-qubit teleportation [9].

The model in Sec. II predicts which observables survive incomplete notices. The schedule and delivery rules in Sec. III allow separate estimates for suppressed replies and for replies missing one field.

II. CONDITIONAL PAULI FRAMES

A. Assignment of the bits

Alice holds the input Q and resource member A. Bob holds B. Start from $|\psi\rangle_Q|\Phi^+\rangle_{AB}$, where $|\Phi^+\rangle = (|00\rangle + |11\rangle)/\sqrt{2}$. A controlled X from Q to A, followed by a Hadamard on Q, maps the Bell measurement onto two local Z measurements. We write v for the result on Q and u for the result on A, with logical value zero corresponding to positive Z. The state on B is $P_{uv}|\psi\rangle$, where $P_{uv} = X^u Z^v$ and a global phase has been suppressed.

Bob measures a Pauli operator σ_j with result $b \in \{-1, 1\}$. If the notice supplies $\hat{u}, \hat{v}$, the sign in the reported frame is given by Eq. (1).

$$b_{\mathrm{f}} = b\,(-1)^{\epsilon_j(\hat{u},\hat{v})}, \qquad (\epsilon_X, \epsilon_Y, \epsilon_Z) = (\hat{v},\ \hat{u} \oplus \hat{v},\ \hat{u}). \tag{1}$$

The order of the two Pauli operations leaves Eq. (1) invariant. Exchanging the printed fields in the decoder would exchange the axes protected by the corresponding one-field policies. We kept the choice of fields fixed from the alignment runs onward.

For a particular decoded notice, set $a = u \oplus \hat{u}$ and $d = v \oplus \hat{v}$. In the ideal circuit Eq. (2) gives the channel averaged over residual syndromes.

$$\mathcal{T}(\rho) = \sum_{a,d} q_{ad} X^a Z^d \rho Z^d X^a, \qquad (t_X, t_Y, t_Z) = \sum_{a,d} q_{ad}\big((-1)^d, (-1)^a, (-1)^{a+d}\big). \tag{2}$$

Here the weights q apply when the residual distribution is independent of the input. With apparatus errors the same expression holds for the extra Pauli channel following the complete-delivery channel if the missing bits are uniform and independent of the conditional apparatus errors. We use that factorization to predict the axis scores under suppression. Raw accuracy is calculated from recorded signs, without inferring independence or inserting a fitted q.

The six inputs are the eigenstates of X, Y, Z. A matching-basis score F_j averages agreement over the two signs, so the average fidelity with an ideal detector is given by Eq. (3).

$$\overline{F}_6 = \tfrac{1}{3}\sum_j F_j = \tfrac{1}{2} + \tfrac{1}{6}(t_X + t_Y + t_Z). \tag{3}$$

Here each t_j is the corresponding diagonal entry of the Bloch transfer matrix. The displacement cancels between the two signs. The six directions have mean zero and isotropic second moment, so this score equals the uniform pure-state average [10, 11]. Estimation of off-diagonal entries requires mismatched-basis data and a measurement model [12, 13].

B. Known omissions

In the return-date policy Bob receives u and fills the empty coverage entry with $\hat{v} = 0$. For an ideal Bell record d is uniform. The residual channel in Eq. (2) is Z dephasing, with $\overline{F}_6 = 2/3$. The missing coverage entry also removes the Y correlator, because either correction bit flips Y. The coverage-only policy leaves X intact. Both policies can give the same overall fidelity while preserving orthogonal observables. In every single-field analysis here, absent values are set to zero before the input sign is available.

Under a known erasure of both fields, the ideal syndrome is uniform over the four Pauli operators before delivery. Averaging over that syndrome makes B maximally mixed. For the erasure fraction

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r, a delivered fidelity F_{d} and an erased score F_{e}, the full-population expectation is the mixture in Eq. (4).

$$F(r) = (1 - r)F_{\mathrm{d}} + rF_{\mathrm{e}}. \tag{4}$$

In the ideal case $F_{\mathrm{e}} = 1/2$. The same erased score holds if Bob's marginal state is independent of Alice's input and the score balances the two input signs, even when Bob's marginal deviates from maximal mixing. Fluorescence errors can then change the marginal fraction of plus outcomes without changing the balanced score. The erased subsets should retain this balance. A transport with delays that depend on the printed text can select on the syndrome itself and invalidate the factorization in Eq. (2), even in the limit of perfect state preparation [14].

III. APPARATUS AND DELIVERY

A. Ion sequence and denominator

The stations are two zones of one segmented trap. We encode three optical qubits in $^{40}\mathrm{Ca}^{+}$, with $|0\rangle$ in $4S_{1/2}$ and $|1\rangle$ in $3D_{5/2}$. The zones share a 729 nm phase reference and use individually addressed preparation beams. We make the resource pair in the common gate zone and separate the ions before loading the input into Q. A timing strobe admits the trial after the trap and laser preflight flags are accepted. This strobe precedes the setting instructions. The local two-qubit Bell gate uses a parity interaction and basis rotations. Measured from the last input rotation to Bob's basis rotation, the run-to-run median elapsed time was 0.68 ms. Alice's electronic Bell values are stored on her controller. Bob's fluorescence result is assigned by a threshold fixed from bright and dark reference runs. Every admitted trial has a result under that threshold, including measurements exactly at the threshold, which are assigned to the bright outcome. The reference runs are outside the denominator.

The controller schedules 24 runs. In each run and for each of four delivery policies, it permutes 100 copies of each of the 18 pairs of input state and Bob basis. Each station receives only its instruction for the current trial. Bob's basis has the same distribution for each input state and the fluorescence record is closed before classical delivery. There are 172,800 admitted trials, with 57,600 matching-basis trials. We retain all of the matching trials for scoring. The three bases and the two signs carry equal weights in each run and policy. Results on the 115,200 mismatched trials serve to check that event alignment and losses do not depend on the scoring mask. The print service does not receive that mask.

The receiver joins scanned notices to a manifest of opaque event identifiers created before acquisition and assigned independently of settings and outcomes. The manifest includes run and policy but omits sender group and preparation. Neither arrival order nor the local acquisition clocks enters Eq. (1).

B. Form and routing

All notices begin with the same sentence, "I am away from the office and will return on the date below." The date field has two fixed-width values, 16 March and 18 March. They encode $u = 0$ and $u = 1$, respectively. The coverage entry carries one of two equal-length directory names, one for each v. They are routing aliases and have no association with the experimental team or the scanner operator. The final line says that correspondence will be read on return. We use the same two dates in all runs, regardless of the days on which the experiment takes place. In an earlier records trial the date rolled over with local midnight [15]. We fixed both values and omitted the day of the week to prevent that rule from changing the sign assignment mid-run.

The formatter receives one request per admitted trial and issues the fields permitted by the assigned policy. Each printed side holds twelve notices. Identifiers are machine-readable and contain no syndrome. Alice sends no electronic correction to Bob. Bob closes the fluorescence log before release of the printed batches and seals the decoded ledger before the preparation log opens.

Table I specifies the four delivery policies. Primary and secondary copies in the split policy share only the event label and fixed form text. The primary notice has the return date, while the secondary notice uses a blank date line and gives the contact during the absence. The decoder reads a single field from each disclosed copy. It supplies the same default zero in the counterfactual one-copy analyses. The two copies follow separate streams, each with its own random order. The analysis treats a missing copy as a missing field, even when its companion carries the same identifier. Similar paired forms were used to verify dispatch acknowledgments in an earlier correspondence experiment [16].

TABLE I. Fields delivered by policy and number of physical impressions for all 43,200 admitted trials in each policy. A dash denotes an empty field or the absence of a second copy.

Policy	Primary fields	Secondary fields	Impressions
Full	Date, coverage	—	43,200
Date only	Date	—	43,200
First per sender	Date, coverage in first reply	—	10,800
Split	Date	Coverage	86,400
Total			183,600

Ordinarily a repeated sender is detected by address and coverage period. In our implementation, the formatter assigns one address to four trials within a run, with time windows that reset at its end. Each of the 18 setting pairs has 25 distinct four-trial groups in the suppression arm. The groups are fixed before the run and consist of trials with identical setting pairs. The first of each group in the ran-

domized trial order gets a complete reply and the other three generate no page entry. This exactly balances both signs in the delivered and erased subsets. The grouping is used exclusively by the suppression service. In particular, Bob cannot infer the input sign from it. The scheduler does not use the Bell outcomes when constructing the groups or selecting their first member.

Every intended notice produced one legible entry. A comparison after scoring found exact agreement with the corresponding electronic source fields. Primary and secondary forms have fixed field positions and are scanned separately. In each layout an unprinted field is a blank, even above a fixed closing sentence. Free-form correspondence would require a different field model [17].

C. Estimation

For preparation sign s_i, the success indicator is $(1 + s_i b_{\mathrm{f},i})/2$. The three-axis mean is a single primary score per run and policy. We report two-sided 95% Student intervals using the 24 run means, treating runs as independent repeats with comparable acquisition conditions. This gives run-to-run uncertainty and allows dependence among trials within an acquisition and printing run. It excludes systematic uncertainty in the basis rotations and binary thresholds. The primary comparisons are the single-policy intervals against $\mathbf{2/3}$. The full-minus-split comparison uses paired differences over the runs. The matching scores are estimates of fidelity under trusted state preparation, fixed basis readout and unbiased transport with respect to the chosen input. Our analysis of preparation and readout errors is in Sec. V.

IV. MEASURED TRANSFER

A. Full and partial notices

The pooled counts in Table II have denominator 4,800 per axis and 14,400 per policy. On full delivery the Z axis has slightly higher agreement than X, and Y has lower agreement than either. This ordering is present with the electronic record and is consistent with phase noise at the qubit level. The printed record was transcribed exactly to the disclosing fields, so the measured difference between full and split is a comparison of two trial populations, with any path-specific readout effects included. Across paired runs the split-minus-full difference is $\mathbf{-0.0025}$, with interval $[\mathbf{-0.0081}, \mathbf{0.0031}]$. The aggregate scores for each retained axis are shown in Fig. 1.

TABLE II. Matches by preparation axis after correction. The final column is the 95% run-level interval for the three-axis fidelity.

Policy	X	Y	Z	Total	$\overline{F}_6$	95% interval
Full	4,548	4,512	4,572	13,632	0.9467	0.9424–0.9509
Date only	2,406	2,388	4,560	9,354	0.6496	0.6428–0.6564
First per sender	2,932	2,936	2,936	8,804	0.6114	0.6045–0.6183
Split	4,536	4,500	4,560	13,596	0.9442	0.9399–0.9484

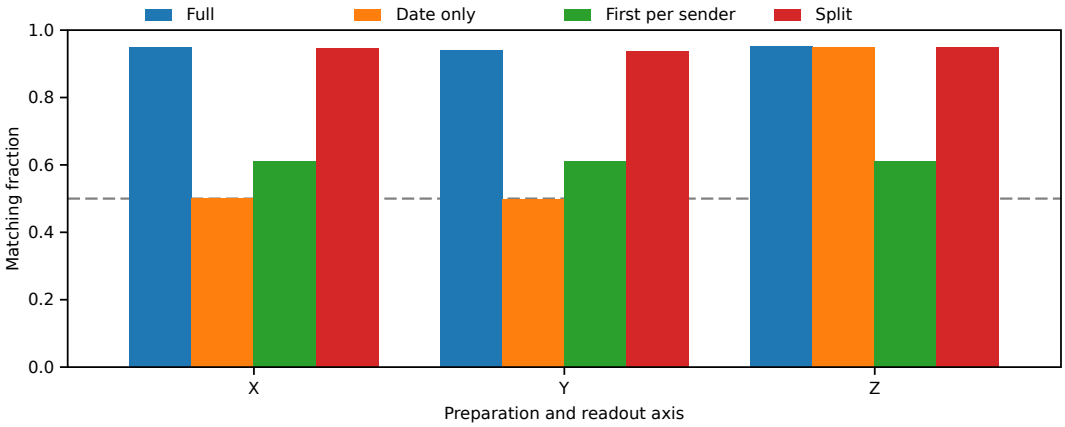


FIG. 1. Matching-basis fidelity by delivery policy. Each bar is a point estimate from 4,800 trials. The dashed line marks chance agreement per axis.

Direct state estimation with a single unknown qubit and classical communication has a uniform-ensemble limit of $\mathbf{2/3}$ [18]. The same limit applies to the six-state ensemble used here, because the measurement-and-preparation score for a linear instrument depends on the first two moments of the input Bloch direction. This limit presumes that the sender receives the input qubit alone and that the receiver lacks both preparation labels and input-dependent side information. Our hardware measurements require trusted controls. When comparing a correction policy with $\mathbf{2/3}$, the score refers to what a measure-and-prepare channel could do with an unrestricted classical message. A smaller bound specialized to a restricted one-bit message would address a different test.

Date-only delivery preserves $F_Z = \mathbf{0.9500}$. The transverse scores are $\mathbf{0.5013}$ and $\mathbf{0.4975}$. Using the observed Z score and $1/2$ for the erased correlators in Eq. (3) gives $\mathbf{0.6500}$, 0.0004 above the measured score. All 14,400 trials have a notice. A readout restricted to Z would miss the difference between one-field and full delivery.

B. Suppressed senders

The first-per-sender policy delivers 3,600 of the 14,400 matching trials. It yields 1,136, 1,128 and 1,144 matches among the 1,200 delivered trials per axis. The corresponding total is 3,408, or $\mathbf{0.9467}$. Among the 10,800 erased trials there are 5,396 matches, or $\mathbf{0.4996}$. We obtain the all-attempt score using Eq. (4), retaining the erased trials at their original weight. In per-sender terms, expecting the same fidelity on each of the four attempts would require a new correction on each attempt. A copy of the first notice can identify the correct sender on a later trial while its two bits remain unrelated to that trial's syndrome.

Fig. 2 shows the predicted dependence on the suppressed fraction r from the full-delivery calibration. The bound for an all-attempt quantum advantage under this scoring rule is $r < 1 - (1/6)/(F_{\mathrm{d}} - 1/2)$, or $r < 0.627$ for the measured full score. Three contacts per sender, of which only the first receives a reply, would exceed this threshold slightly ($r = 2/3$). Two contacts per sender would meet it if the delivered score and missingness model stayed fixed. The experiment tested the full and four-contact rules on separate trials. The intermediate scores are predictions of the mixture model.

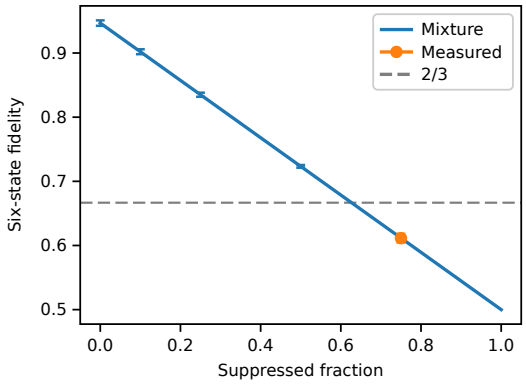


FIG. 2. Fidelity versus fraction of suppressed records. The curve uses full-delivery fidelity and erased score 1/2. The bars on the curve propagate the 95% run interval for full delivery. The point at r = 0.75 is the observed first-per-sender score with a 95% run interval.

The tolerance depends on the correction that disappears. If repeated requests lost only the coverage line, the mixture would contain a dephasing channel whose Z score remains high. Encoding the phase correction in one of the otherwise fixed lines could change that outcome. Our formatter draws a fixed salutation and closing from the account. It can choose the contact and date for each response. The implementation of the first-per-sender rule suppresses the response before field selection. Moving a bit from the coverage line to a different line on the same sheet would therefore leave Fig. 2 unchanged under the tested policy.

C. Separate recipients

The split response provides a direct within-trial test of the frame assignment (Fig. 3). With the primary copy alone we find 2,396, 2,411 and 4,560 matches. With the secondary copy alone we find 4,536, 2,412 and 2,382 matches. Both ledgers contain all 14,400 identifiers. The full score returns when the corresponding notices are brought together. Because the three analyses use the same fluorescence data, their differences come entirely from the classical decoder. The Z result from the primary alone equals the joint Z result trial by trial. The X result has the same property for the secondary alone. On Y, a wrong sign in each of the two single-copy scores can be correlated, since the full sign uses the parity of the missing corrections. We did not treat the one-copy scores as independent samples.

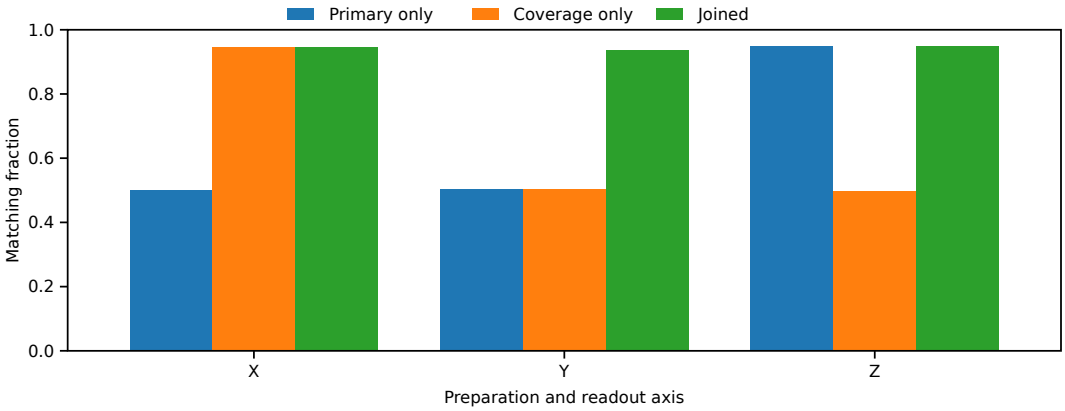


FIG. 3. Results of decoding the split-policy readout from the primary notice, the coverage notice, or their join. All three estimates use the same 4,800 measurements per axis.

In this scheme a correspondence clerk can recover the phase bit by looking at the secondary notice, which is filed under the temporary contact. The primary copy alone, although sufficient to locate the intended time of return, cannot restore phase. The two copies together need neither an ordering rule for the dates nor a decision about which recipient has jurisdiction over the state. They do require a unique event identifier. The standard follow-up procedure of filing by address alone would join records from different trials if used with the four-request groups in Table I. That error is of order unity for Y. Increasing the window during which the address is treated as the same sender would make the association more ambiguous and would not recover the missing terms.

V. ERROR BUDGET AND APPLICABILITY

The observed full-delivery infidelity is 5.33×10^{-2}. In separate electronic calibrations we estimate the contribution associated with resource preparation and transport of the trapped ions at order 2×10^{-2}, including the finite contrast of the Bell resource after separation. Bell gate and local mapping errors are of order 2×10^{-2} from parity and Bell-basis calibration sequences. Preparation and readout give a combined order of 10^{-2}, based on reference sequences with no teleportation gate. These are operational estimates with overlapping uncertainties. An independent sum would neglect shared errors in the laser phases and in the axis calibrations. We leave the reported scores uncorrected. At this precision the same device error that lowers the complete-delivery fidelity can lower the Z fidelity of the date-only protocol while leaving its transverse scores within sampling error of $1/2$.

Finite-ensemble scoring removes the need to reconstruct an entire process before quoting the average fidelity. The price is that coherent rotations and channel anisotropy can enter the same diagonal estimate. A rotation-axis or phase-reference error can appear in the score as imperfect recovery even if the decoded record is correct. Rerunning the analysis with the archived electronic result does not

remove such an error. It tests the printed route. A second receiver apparatus would require an independent estimate of the relative measurement frame.

The tests used two date values and two contacts. With other forms, a forwarded or reprinted response can replace an empty field with a dash. A parser that rejects the replacement reports an erasure. A parser that accepts it as the other valid value produces an unflagged Pauli error, despite successful delivery.

The storage interval of interest here is the time for which a receiver needs a corrected state. In a Pauli measurement protocol, Bob can act before delivery if he records the requisite measurement basis and preserves its outcome. Applying the same procedure to a subsequent adaptive non-Clifford operation generally requires the frame to arrive first, or a protocol designed to defer the dependency [4, 5]. For live state transfer the memory budget would have to include the delay of notices deferred until the following dispatch.

VI. CONCLUSION

A delivery policy is part of the specification of a teleportation channel if it can change which Bell record reaches the receiving frame. For the forms studied here, the absence of a coverage name can be detected in quantum measurements, even when the printed return date and event identifier are present. Recombining copies is sufficient when they retain the two complementary fields. A more stringent test of an absence service would route a state through two consecutive coverage assignments. Its prediction begins with the Pauli parity of both records, so a blank field in either assignment could remove a correlator even if the final recipient gets a complete notice.

APPENDIX A: SINGLE-QUBIT CLASSICAL BENCHMARK

A measure-and-prepare channel can be written with effects $M_k = w_k(I + \boldsymbol{m}_k \cdot \boldsymbol{\sigma})$ and outputs of Bloch vector $\boldsymbol{n}_k$. Completeness gives $\sum_k w_k = 1$ and $\sum_k w_k \boldsymbol{m}_k = 0$. For a pure input with Bloch vector $\boldsymbol{r}$, outcome k occurs with probability $w_k(1 + \boldsymbol{m}_k \cdot \boldsymbol{r})$. Its reconstruction has overlap $(1 + \boldsymbol{n}_k \cdot \boldsymbol{r})/2$. Averaging over the six directions used in Eq. (3) gives Eq. (5).

$$F_{\mathrm{mp}} = \frac{1}{2} + \frac{1}{6} \sum_{k} w_k \, \boldsymbol{m}_k \cdot \boldsymbol{n}_k \le \frac{2}{3}. \tag{5}$$

Positivity imposes $|\boldsymbol{m}_k| \le 1$, and $|\boldsymbol{n}_k| \le 1$ follows from positivity of the output state. Equality is attained by measuring a fixed Pauli operator and preparing its measured eigenstate. In Eq. (5) the receiver has no setting-dependent timing or sender metadata. The matching mask stays private until scoring.

APPENDIX B: RUN-LEVEL COUNTS

Each cell in Table III counts correct matching-basis decisions out of 600. To reconstruct Table II, for a given policy let m_i be the number in run i divided by 600. The point estimate and interval follow from Eq. (6).

$$\frac{1}{24} \sum_{i} m_i \;\pm\; t_{23,0.975} \sqrt{\frac{\sum_i (m_i - \overline{m})^2}{24 \cdot 23}}. \tag{6}$$

For the paired contrast replace m_i with $(N_{\mathrm{split},i} - N_{\mathrm{full},i})/600$. Using a binomial standard error on the full pool would remove the effect of any common shift in the readout or sorting of a run. Most such shifts should cancel from the paired contrast if they affect all policies equally. The scatter visible in Table III is sufficient to use Eq. (6) without assigning a separate variance to the printer.

TABLE III. Per-run correct matches used in the interval estimates. Run order is acquisition order.

Run	Full	Date only	First per sender	Split
1	569	397	372	572
2	574	389	368	559
3	571	383	366	565
4	564	397	357	572
5	565	378	371	564
6	571	401	362	556
7	575	380	354	577
8	570	400	384	569
9	558	377	377	561
10	559	402	365	562
11	572	374	372	571
12	576	406	382	561
13	562	389	351	564
14	563	390	359	574
15	563	399	369	574
16	573	387	370	561
17	577	393	364	572
18	567	382	379	575
19	566	405	374	559
20	565	381	361	565
21	573	390	350	568
22	578	386	380	568
23	560	393	354	558
24	561	375	363	569

APPENDIX C: FIELD DICTIONARY

The date decoder matches the full date string, including the spacing after the number, to a table with two rows. For the coverage field, the corresponding table contains the two display names. Either blank is an erasure. The title of the contact and the signal that the sender will read mail on return are fixed template text. The recipient identifier and mailing class are stored as part of the delivery ledger, outside the two-bit alphabet. The lookup for a split response is keyed by trial identifier and class. A detached coverage slip without its trial identifier counts as an erasure.

The total in Table I corresponds to 15,300 printed sides at 12 impressions per side. Distinct recipient classes need distinct bins during scanning. One run yields 3,600 split impressions, divided equally across the two classes without unused positions. The 450 impressions from a first-per-sender run would leave six empty positions on its last printed side if runs were printed separately. In our count consecutive runs of that policy share the last printed side when necessary. The run mark disambiguates the two sets on that side.

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