capstract:2611.00013v1 [eess.SY] 6 Nov 2026

Regulation of Diagonal Rocking Under Accession-Order Constraints

Inés A. Verdual
Automatic Control Group
Instituto Técnico de la Vega del Cerdán
Zaragoza
Spain
i.verdual@cerdan.example

Eero S. Niskaro
Department of Precision Mechanics
Kaarne College of Applied Measurement
Turku
Finland

Leonie T. Bosmare
Collections Engineering Section
Rijnhaven Institute for Material Records
Leiden
Netherlands

***Abstract*—A four-point support admits a signed measure of rocking that changes by one sheet thickness when a sheet enters any corner stack. We study a reading table supported on four subject files. Every new library accession supplies a card and fixes its destination. The controller can choose only the order of filing from a finite incoming tray, with a deadline for each card. We give a prefix bound on the available correction and an instance-specific lower bound from the compulsory and eligible accessions. In sixteen balanced lots of 80 entries, chronological filing produced a mean lot root mean square gap of 0.968 mm. Selecting the next card by measured gap reduced this to 0.290 mm with eight places in the tray. A run of purchases on a single diagonal accounted for 37 of 110 excursions beyond 0.5 mm at that capacity. Four lots repeated on an instrumented table gave an average root mean square gap of 0.213 mm with eight places. All policies end each lot with the same distribution of cards. The acquisitions available before closing determine how long a correction can be held.**

***Index Terms*—constrained control, discrete-event systems, disturbance rejection, reordering buffers**

I. Introduction

A reading table with four nominally equal legs rocks on a floor whose corner heights do not lie in one plane. The usual correction adds a small thickness to one of two feet. When the material available for correction has already been assigned to a particular foot, the choice is more limited. In a reading room fitted with four accession files, one under each foot of the central table, a new volume of history raises the history foot. A second copy of the accession card would produce a second catalog entry. The operator may hold the card on the desk until another subject arrives.

Our unit of time is one accession. A locally prepared list gives each volume one of four subject codes. Each code corresponds to a separate shallow file whose top surface supports a foot. The active reading copy of the catalog is held elsewhere. These cards form a deposit set and may accumulate under load until the annual transfer to boxes. The registration ledger carries the date on which a card enters the room. A deposit may follow registration by a few entries, provided an older item is never kept beyond its allowance. The question is how much of a geometric error can be taken out using this interval between registration and deposit.

The controlled variable has a sign. A play of 0.6 mm at the northeast corner calls for the opposite diagonal from a play of 0.6 mm at the northwest. Consecutive accessions in literature and science both raise the same diagonal in our installation. A substantial improvement in the catalog’s subject coverage can therefore coincide with a loss of regulating authority. We first derive the mechanical and ordering constraints. We then replay exactly the same purchases with different capacities on an ideal support and reproduce a subset with a physical card file. Results distinguish errors due to the order of purchases from errors due to the finite flexibility of the tabletop.

II. Related systems

A linearized plant with a measured output disturbance and two equal, opposite input quanta would ordinarily admit a relay controller [1], [2]. Here the signs on the quanta belong to individual objects. Selecting a positive quantum changes the set of quanta available at the next instant. Models of switched dynamics [3] and event-driven state descriptions [4] accommodate that dependence if the state includes the tray. Ignoring the tray yields a small control model with a physically unavailable input. A test of such a model on independent signs would miss both the drain at the end of a lot and the effects of a long batch from one vendor.

Reordering buffers generally admit an item, hold it, then release it to reduce some sequence-dependent cost [5], [6]. The cost in this experiment accumulates with the *sum* of signs deposited so far. It is possible to have two histories ending in the same subject and differing by several millimeters in their next required correction. A full dynamic program could include the

deposited counts [7]. Terminal constraints in predictive control can restrict which finite-horizon solutions remain admissible after the horizon advances [8]. We use a one-step controller and a bound that does not assume that this controller is optimal.

Card caliper in a compressed pile depends on moisture and on the relief around the print impression [9]. Earlier work on removable machine feet measured settlement after a single insertion [10], and file conservators have measured permanent shortening after closed storage [11]. The deposit file undergoes both changes. During the short runs reported here an increment measured under the foot load describes the gap reasonably well. The pile continues to creep during a long closure. We have no observations over the annual transfer interval.

III. Reading table and input

Index the feet A, B, C, D clockwise around a rectangular top. At a fixed clock tick let z_i be the height offered to the underside of the table by support i, including its file. Opposite corners of any plane above a rectangle have equal height sums. A signed departure from that plane is

$$x = z_A - z_B + z_C - z_D. \tag{1}$$

If three rigid feet make contact, the fourth-corner discrepancy has magnitude $|x|$. A force alternately applied over each diagonal reveals the orientation of the gap. The top need not be horizontal when $x = 0$. An affine floor slope cancels from Eq. (1). A spirit level on the center of the table is consequently a poor instrument for this measurement.

The subject assignment in Table I is fixed for the life of the experiment. A stack rises by h for each deposited card. With d_k the floor contribution at tick k and s_j the sign of accession j, the output after filing at tick k is

$$x_k = d_k + h \sum_{j \in F_k} s_j, \tag{2}$$

where F_k is the set filed through that tick. Exactly one new element enters F_k at each tick. A cataloger cannot assign a different sign to a book of botany after observing x. For subjects spanning two files we retain the primary subject as it stood on purchase. Adding a cross-reference slip would change the input supply and the mechanical model.

TABLE I
Subject destinations viewed clockwise from the reader’s left. Each lot deposits the same terminal count at each foot.

Foot	Subject	Sign	Cards per lot
A	Literature	+1	20
B	History	−1	20
C	Sciences	+1	20
D	Arts	−1	20

A. Release and deadline

A tray has B places. At tick 1 the first B registrations are visible. After each deposit one further registration becomes visible until the source lot is exhausted. Thus at tick k the largest usable accession is $u_k = \min(N, k + B - 1)$. When accession j is still in the tray at tick $j + B$, the operator must deposit it then. By induction all smaller accession numbers have already met their own deadlines. A single forced release per tick suffices, and chronological order is always feasible. The last deposit at tick N empties the tray even if a card still had time on its allowance.

This interpretation of capacity includes the position at the filing desk. For example, with two places one may file accession 2 at tick 1. At tick 2 one may file 3, leaving 1 alone with 4 at tick 3. At tick 3 accession 1 is due. An implementation which counts the card already in the cataloger’s hand as a third available card gives a different result for the same label on the tray. We inspected the registration number and the clock in every run. No reconstructions of the queue from the accumulated stack heights were required.

The input to the regulator is a measurement of the current gap after the prescribed floor movement and before the new card enters a stack. The controller subtracts nothing for expected future motion. It predicts the result of each candidate by adding its signed caliper. At a deadline it takes the compulsory accession. Otherwise it minimizes the absolute predicted residual, breaking ties by the smaller accession number. Two volumes from the same diagonal have identical one-step mechanical scores. This tie rule can therefore affect later availability although it cannot affect the gap at the current step. The mechanical trial uses the measured gap on each tick and the same rule.

IV. Bounds imposed by the catalog

Let $C_k = \{1, \dots, k\}$ be the chronological prefix. Every policy depositing k cards must exchange equally many members of C_k for accessions after k. There are at most $B - 1$ later accessions available at that tick. At the beginning and end of a lot there may be fewer. Since the sign difference on exchanging one card is at most two, a policy satisfies

$$|x_k - x_k^{\mathrm{chron}}| \le 2h \min(k, B - 1, N - k). \tag{3}$$

The right side of Eq. (3) vanishes at the final deposit. For a long imbalance in the original purchase list it gives a simple obstruction. A correction of q mm relative to chronology requires at least $\lceil q / (2h) \rceil$ visible places in addition to the front card. The additional places must also contain cards of a useful sign. There is little reason to build another place into a tray whose entire contents are period histories of the same country.

We can obtain a tighter bound from the actual headings. At tick k all indices up to $l_k = \max(0, k - B)$ have reached their deadlines. Write S_l for the sum of these signed cards. The remaining $m = k - l_k$ deposits must come from the interval $l_k + 1$ through u_k. Sort the signs in that interval from smallest to largest. If L is the sum of the first m and H the sum of the last m, an outer interval for the signed gap is

$$I_k = [\, d_k + h(S_l + L),\; d_k + h(S_l + H) \,]. \tag{4}$$

Its distance from zero gives a lower bound for the absolute residual of *any* admissible controller. The interval is continuous, while the reachable signed sums have parity k and lie on a grid of spacing $2h$. A grid correction tightens the bound when the interval contains zero. These are pointwise necessary conditions. Choosing the closest grid point independently at each tick can yield a sequence in which a card already in one foot would have to be moved to another. We do not take the sequence of closest points as a feasible controller.

The compulsory prefix in Eq. (4) includes the card due at the current tick. The bound therefore applies after that tick’s deposit. Computing it before appending the card can understate the residual by a full exchange of signs. We give a numerical instance in Section VI.A for which one can verify the loss of authority directly from 15 eligible catalog entries.

V. Excitation and endpoints

Each of sixteen lots contains 80 accessions with equal counts in the four subjects. We used a balanced order before shuffling, then allowed unrestricted runs. The archival number is the position in this shuffled order. The four allocated subject budgets at a small branch commonly close at similar counts, but actual batches can contain unequal counts, particularly at the end of the financial year [12]. Balancing by design permits a comparison in which the terminal support heights agree exactly. The early prefix can still contain many works on one diagonal. Every capacity sees the same purchase list and the same floor trajectory for a given lot.

For lot b, numbered from 0 to 15, the imposed motion in millimeters is

$$d_k = 0.8\sin(2\pi k/40) + 0.4\sin(2\pi k/16 + \pi b/8). \tag{5}$$

The longer component drives two complete periods during a lot. The shorter one drives five, with a different starting phase in each lot. Eq. (5) specifies a disturbance just before deposit, not an elapsed-time model of a floor in a working library. It gives the catalog several opportunities to need the same diagonal in a day. We fixed $h = 0.200$ mm and tried capacities 1, 2, 4, 8 and 16. The chronological comparison is the same model with $B = 1$. Arrivals stop at entry 80. Scores include the drain from a full tray to a closed file.

We defined the lot score as the square root of the mean of its 80 post-deposit squared gaps. We report the arithmetic mean of the sixteen lot scores. A second endpoint counts clock ticks strictly beyond 0.500 mm. For equalities at the boundary, numerical comparisons use a tolerance of 10^{-10} mm. A 0.500 mm gauge used by the mechanical technician motivated this endpoint. We did not calibrate it to readers’ willingness to report a wobble. The numerical lot and excitation pairs form a fixed test set. We report their spread and extrema without a sampling interval for the population of all acquisitions [13], [14].

A. Physical repeat

A rectangular frame supported a 620 by 940 mm reading surface at four machined lands. Four removable file cartridges carried the top plate load. We mounted foot A on a translation stage so that the prescribed floor motion entered the expression in Eq. (1) once, with positive sign. A two-position loading head shifted a small test load alternately across diagonals. A displacement probe above the test corner read the difference. Calibration against a series of temporary corner spacers gave the signed equivalent corner gap. Two calibration passes covered the whole motion range from each side [15]. For excursions through zero, reversal of the loading head fixed the sign before the next card was selected.

The technician presented cards in arrival order from a numbered packet. A separate operator read the registration number selected by the controller, inserted the card at its subject foot and recorded the new gap. We used the first four lots at capacities one and eight, a choice made before inspecting the model output. The frame was unloaded between lots, all four cartridges emptied and the same card packet returned to arrival order. Half the lots ran first in chronology and half first with eight places. Time to read a label did not advance the disturbance. This enabled both trials of a lot to receive the same floor motion.

A separate stack of 20 blank cards from the same stock gave 4.01 mm under the test load after one minute. The same stack read 4.12 mm held just between the micrometer faces. We used the rounded loaded value of 0.200 mm in the controller. The typed heading was located outside the small bearing patch, which limited the effect of differing amounts of type. A card with a turned corner was replaced from duplicate deposition stock before numbering the packets. The number assigned to that accession likewise appeared only once in the reading-room file. During the trial the serial number was visible in a window beside the support land, so no foot had to be unloaded to check compliance.

VI. Results

The change for each capacity appears in Table II. In 1,280 chronological deposits the post-deposit gap exceeded half a millimeter 736 times. With eight places the count was 110. A capacity of eight reduced the lot score in all sixteen matched comparisons. It still gave a mean score above a full card thickness, principally because of two runs with little stock on the required diagonal. The width-16 trial had no excursions over the specified gauge. A wider tray also allowed more accessions to move away from their chronological position. We counted a displacement of either sign as one such move.

TABLE II
REPLAY ON SIXTEEN LOTS, 1,280 DEPOSITS PER CAPACITY. THE SCORE COLUMNS GIVE THE MEAN AND LARGEST OF THE SIXTEEN ROOT MEAN SQUARE GAPS.

Places	Mean score (mm)	Largest score (mm)	Ticks above 0.5 mm	Accessions moved	Deadline releases
1	0.968	1.758	736	0	0
2	0.855	1.658	609	736	138
4	0.629	1.452	394	1,042	127
8	0.290	0.937	110	1,153	48
16	0.141	0.180	0	1,168	0

At width one every release follows from availability alone. The compulsory-deadline counter stays at zero because the single front card is filed before its deadline. At width two the first compulsory release can come at the third registration. With eight places there were 48 deadline releases in the entire set, even though over a thousand cards changed their filing tick. Most displaced cards had been filed early or had been kept a few ticks and selected for their sign. The counts in the last two columns would answer quite different questions about the workload of a cataloger.

Fig. 1 shows sampled ticks for lot 0. The two policies arrive at the same final gap after visibly different excursions. In this lot the floor disturbance also happens to pass through zero on the last tick. In lot 4 the final gap is 0.400 mm for every capacity. That residual exists with exactly 20 cards at every foot. The floor is off plane when the last card has to be filed and the controller has no remaining choice of support.

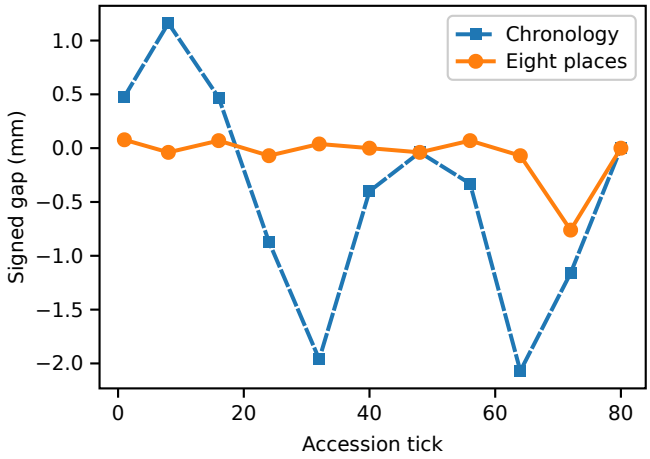


Fig. 1. Post-deposit gap at selected ticks in lot 0. Lines connect the sampled ticks. Scores use all 80.

A. Subject imbalance

Lot 4 supplied 37 of the 110 half-millimeter exceedances with eight places. Around tick 46 the acquisition order contains a long excess of literature and sciences. At that tick $l = 38$, $u = 53$ and $m = 8$. By taking the eight most favorable of the 15 optional signs, a controller could bring the deposited signed sum down only to six. The corresponding upper limit is 16. At a 0.930 mm floor offset the compulsory minimum gives $0.930 + 6(0.200) = 2.130$ mm. The actual residual was 2.130 mm to the precision shown in Fig. 2. At tick 47 the smaller bound was 2.082 mm. Giving the controller a forecast of tomorrow’s books cannot improve its score at either tick under the same accession rule.

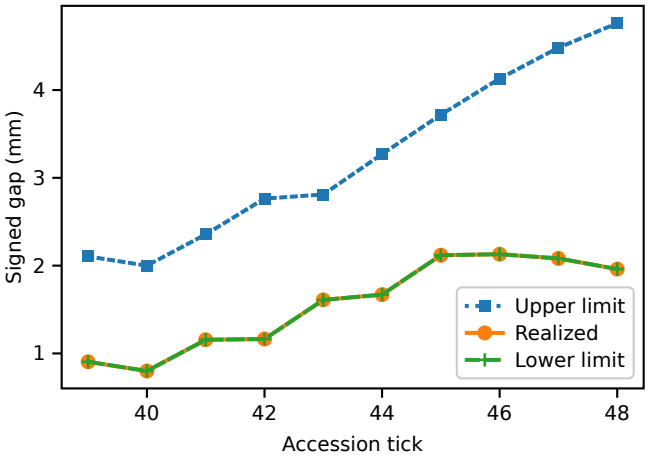


Fig. 2. Allowable interval for the signed gap near the largest excursion of lot 4. Measured in replay, the realized gap coincides with the lower endpoint at each displayed tick.

This diagnosis can be made without disclosing individual titles. A signed count of compulsory accessions and a count of positive signs in the optional window determine the interval. Conversely, a record of the net number of books in each discipline at closing is nearly uninformative about the mid-lot gap. All sixteen lots have the same four numbers in the closing report. If records are to be used for acceptance tests on the file regulator, the sequence of primary headings must travel with the totals.

B. Mechanical repeatability

Table III gives the paired physical runs. Across the first four lots, ideal eight-place scores averaged 0.191 mm and measured scores averaged 0.213 mm, rounding from the displayed lot values. The difference was largest in lot 1. On that lot the probe changed sign near zero on two occasions when the model still predicted a small positive residual. Once different cards have been taken from the tray, subsequent measurement and model errors cannot be separated by subtracting the two *closed-loop* time series. We therefore also ran the model using the actual deposited signs. Over the 320 eight-place observations the root mean square measurement residual in this second comparison was 0.047 mm.

TABLE III
LOT ROOT MEAN SQUARE GAPS (MM) WITH IDENTICAL INPUT PACKETS. EACH ENTRY USES 80 SETTLED DEPOSITS.

Lot	Chronol. model	Chronol. bench	Eight model	Eight bench
0	1.083	1.094	0.223	0.239
1	0.727	0.745	0.186	0.222
2	0.742	0.751	0.158	0.177
3	1.006	1.019	0.196	0.214

The performance distribution over the full purchase set in Fig. 3 is considerably wider than a bench repeatability interval. The bars show one sample standard deviation over lots. In particular, the small spread at width 16 is a property of these balanced, short lists. It cannot be transferred to a year in which the sciences arrive by standing order and the art folios arrive in a single consignment. We did not draw repeated randomizations until an extreme prefix appeared. The scatter at a given capacity includes exactly one shuffle for each of the specified phase settings.

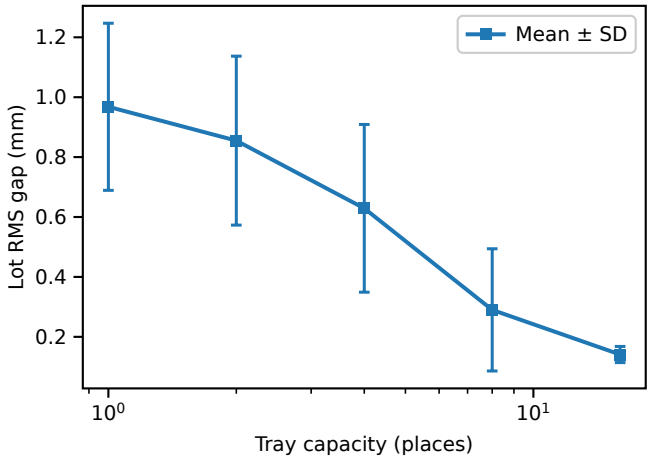


Fig. 3. Mean of lot root mean square gaps, with one sample standard deviation between the sixteen lots. Tray width is plotted on a log scale.

VII. DISCUSSION

A second card on the required diagonal can lower the gap even if its nominal subject differs from the first. This equivalence follows from the geometry in Eq. (1) and has no bibliographic counterpart. Literature and sciences may share a mechanical inventory while retaining separate files. A recent influx of scientific monographs cannot be used to cover a shortage of science cards in the catalog itself. It can, during the period before filing, cover a shortage of literature under the other positive foot. Keeping the file labels visible prevented the operator from making a permanent substitution.

The result also depends on the unit being deposited. An accession slip printed on a heavier stock supplies more than one nominal increment. An archive with mixed stock could store a measured caliper beside each pending number and rank the actual candidates. The two signed counts used for Eq. (4) would then need the ordered signed calipers, although the equal-card exchange argument still yields a bound in terms of the thickest card. A bound on an entire lot would need a measurement under load for a mixed stack. The stock used for music scores in the lending collection was thick enough to show a separate ridge in a mixed pile. We excluded it before registration of the test lots [16].

The table inevitably rises as the controller works. At the end of every balanced replay each corner has gained 4.0 mm and the gap attributable to cards is zero. Correction of the common height would require removing the same thickness from the four feet, an operation reserved for the transfer of a complete deposit set. Our controller cannot make a table lower. A long-horizon performance assessment should include the common rise if the surface has to fit under a fixed reading lamp [17]. Monitoring the signed gap alone would classify a stack of several annual catalogs as satisfactory whenever all the subject budgets happened to balance.

The forced-release rule was enforced even when placing a due volume increased the gap. A room with a more permissive deadline could store the same number of cards for longer, but any long run of registrations of one sign still runs into Eq. (3). An alternative practice sometimes proposed is to assign one classification to the deposited copy and a different classification to the reader’s copy. These trials credit a filing action only when both copies agree. Allowing a different destination would alter the input alphabet and require a different catalog.

VIII. CLOSING OBSERVATION

At the return of the physical packets, we counted the cards and measured the four support heights once more. A reader inspecting the closed files could recover the last signed error from the floor measurement and those four heights. It took the registration ledger, with its intermediate deposit numbers, to determine whether the table had rocked while the books were coming in.

APPENDIX A. REPLAY DETAILS

To construct lot b, start with the ordered string ABCD repeated 20 times. Shuffle its 80 entries in place with the Python 3 random module’s Random instance initialized by the integer $1103 + b$, using its default shuffle. The resulting positions 1 through 80 are accession numbers. For relative-error ties we compare the two magnitudes after rounding to ten digits after the decimal point in millimeters. This convention gives the earlier registration priority when a mathematically exact passage through zero leaves a small trigonometric evaluation error. For the count above the half-millimeter threshold the comparison is against 0.5000000001 mm.

The first eight headings of lot 0 are C, D, C, C, C, D, B and A. Its fourth and seventh registrations could both appear in a tray of width eight before a reader had received the first volume. In that trial the first eight *filed numbers* were 2, 6, 7, 9, 10, 1, 12 and 3. After the first five deposits every chosen sign was negative, so the cards supplied a net correction of −1.000 mm against a floor offset of 0.935 mm. The next offset was

0.930 mm. Measured before that sixth deposit the nominal gap was negative and the earliest positive candidate was registration 1. This short prefix checks the event clock as well as the tie rule.

Appendix B. Mechanical scale and gauge conversion

Place three vertices of the rectangle on a plane. The plane height at the fourth vertex is the sum of heights at its two neighbors minus the height at the opposite vertex. Subtraction from the observed height gives either x or $-x$, depending on the chosen vertex. This identity yields Eq. (1) without an assumption about floor pitch. In the physical frame a load on the diagonally opposite edge bends the top slightly. The probe displacement away from the contact point therefore needs a calibration gain to report the equivalent corner play [18]. The independent spacer runs produced gains of 1.018 and 1.021 on the two loading directions. We used their mean for all eight physical runs.

Settlement was timed from restoration of the foot load. At ten seconds the change over the next five seconds was below 0.010 mm for both reference stacks. We used the reading at ten seconds in Table III. A technician could easily shorten a lot by alternating probes after every deposit, but this would give systematically younger cards to one loading direction. The terminal check allowed a full minute with the plate centered. It concerns the slow common rise of the four stacks and was excluded from the 80 clocked observations in each run.

Acknowledgments

The deposit staff at Rijnhaven supplied duplicate cards with the same stock and type impression as the reading-room set. Asta Korvinen made the spacer measurements before the purchase packets were opened.

References

[1] K. J. Åström and R. M. Murray, *Feedback Systems: An Introduction for Scientists and Engineers*. Princeton University Press, 2008.

[2] Y. Z. Tsypkin, *Relay Control Systems*. Cambridge University Press, 1984.

[3] D. Liberzon, *Switching in Systems and Control*. Birkhäuser, 2003.

[4] C. G. Cassandras and S. Lafortune, *Introduction to Discrete Event Systems*. Springer, 2008.

[5] M. T. Rendalen and N. Ciravel, “Sequence costs in a finite input tray,” *European Notes on Discrete Operations*, vol. 9, pp. 61–78, 2017.

[6] T. A. Kesselro, “Release deadlines in single-server rearrangement,” *Transactions on Event Scheduling*, vol. 17, pp. 119–136, 2020.

[7] D. P. Bertsekas, *Dynamic Programming and Optimal Control*. Athena Scientific, 1995.

[8] D. Q. Mayne, J. B. Rawlings, C. V. Rao, and P. O. M. Scokaert, “Constrained model predictive control: Stability and optimality,” *Automatica*, vol. 36, pp. 789–814, 2000.

[9] A. M. Suvilo, “Caliper measurements of printed card stock under repetitive compression,” *Journal of File Materials*, vol. 24, pp. 23–38, 2019.

[10] P. Ebraldi and L. Veermanis, “Settlement following insertion of removable corner pads,” *Quarterly of Support Metrology*, vol. 6, pp. 49–67, 2021.

[11] M. Dorsenne and O. S. Nilvar, “Compression set in closed subject files,” *Proceedings of the Record Materials Circle*, vol. 11, pp. 90–104, 2016.

[12] J. C. Virelas, “Serial dependence in the receipt of balanced branch acquisitions,” *Collection Accounting Review*, vol. 13, pp. 71–93, 2022.

[13] R. G. Sargent, “Verification and validation of simulation models,” *Journal of Simulation*, vol. 7, pp. 12–24, 2013.

[14] A. M. Law, *Simulation Modeling and Analysis*. McGraw-Hill Education, 2015.

[15] L. Morrenstam and C. Cerdeil, “Directional errors of opposed contact gauges on rectangular frames,” *Bench Measurement Letters*, vol. 12, pp. 107–118, 2018.

[16] M. P. Elvane, “Loaded height variation across media in music accession files,” *Yearbook of Collection Materials*, vol. 8, pp. 15–28, 2020.

[17] I. Tammelov and R. Brisetti, “Clearance budgets in reading-room furniture with replaceable supports,” *Studies in Archive Engineering*, vol. 5, pp. 73–88, 2023.

[18] S. V. Jormela, “Equivalent corner gap in a flexural plate fixture,” *Transactions on Dimensional Fixtures*, vol. 22, pp. 145–160, 2017.