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Article

Free drainage preserves the order of precedence at an unoccupied table

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Where several places receive a toast from one vessel, the head at the first place has drained longer than the head at the last. We tested whether this difference survives removal of the people, the place cards and the record of the pour. In 48 six-place rehearsals a single exposure of the aqueous foam at the table recovered 679 of 720 pairwise precedences; it recovered the complete table in 19 rehearsals. A second set of rehearsals allowed time for an accredited but absent delegate without putting a glass at that place. Of four possible interior positions, the vacant office was inserted correctly in 30 of 48 trials. Equalizing the ages of the glasses reduced insertion to 11 of 48. Liquid content measured across a narrow band of the foam accounts for most of the ordering; changes in the height of the head give poorer resolution. The readout applies while the time between pours exceeds the uncertainty in the drainage age and while the prescribed order is followed.

The official record of a formal toast ordinarily identifies the speakers. It seldom gives the order in which the glasses were filled. For several ceremonies the distinction matters. The toast is simultaneous, yet the table is served in order of accreditation, and a pause of one service interval is observed for a delegation whose seat has been retained in its absence[1,2]. Seating diagrams alone do not always resolve a dispute. A place near the presiding officer can be assigned to a translator and an honorary place may lack a glass. In the Veslan service books two different seniority lists are accompanied by the same drawing of the table[3]. One could settle the question if the preparation of the glasses left a readable trace in a photograph taken at the toast.

A freshly made head of foam provides several possible traces. Liquid drains down the channels between bubbles; gas crosses films and changes the distribution of bubble size; occasional films break[4–6]. On the time scale of a small table it is the liquid that moves furthest relative to its initial distribution. The first place served has the driest head at the instant of the toast, given equal starting conditions. Inverting this statement is more demanding. A senior place can receive wetter foam to start with, two places can be served from different jugs, and the head can change height while the camera is exposing. We tested an inference based on one frame of six heads under measured differences in filling. At a second table five heads had to account for six offices. The instrument was required to give the absent office a rank without being told who occupied any seat.

A material time for the upper part of the glass

Our vessels have a narrow planar window through which the wall borders of the foam can be photographed. An insulating mask covers everything except a band 6 mm high centered 20 mm above the initial liquid surface. The wetted junctions between bubbles and wall are brighter than the interiors in reflected light. We measure the area fraction b occupied by the bright regions. Thickening a wall border increases b even when the gas fraction and the height of the head are too similar to distinguish by eye. Because wall wetting depends on the condition of the window, b must be converted to an equivalent interior liquid fraction f for this apparatus. We used conductance at the same height as a secondary standard and checked it against gravimetric samples in sacrificial cells. The relation is empirical throughout our analysis[7,8].

A separate calibration consisted of 36 columns prepared on six mornings and followed from 10 to 120 s. The upper band lost liquid most rapidly during the first half minute. Individual curves rarely crossed after 20 s when put on the same initial scale, but their initial levels varied enough that the age of one unmarked vessel could be wrong by several seconds. Over the subsequent service interval the mean local liquid fraction followed

$$f(t) = \frac{f_0}{1 + t/\tau}, \qquad u(f) = \tau \left(\frac{f_0}{f} - 1 \right). \tag{1}$$

Here $t = 0$ at the end of filling, u is an apparent age in seconds, and $f_0 = 0.180$ is an extrapolated initial liquid fraction for the band. The fitted time τ was 64 s. A smooth monotone fit and equation (1) give exactly the same ranking within a frame; a numerical age is needed only for the missing-place experiment. We fixed the two parameters before receiving the table images. The decay in Fig. 1 is a local fit and carries no claim about the long-time limit of the foam. Redistribution between horizontal slices and outflow to the liquid below both affect f[9,10].

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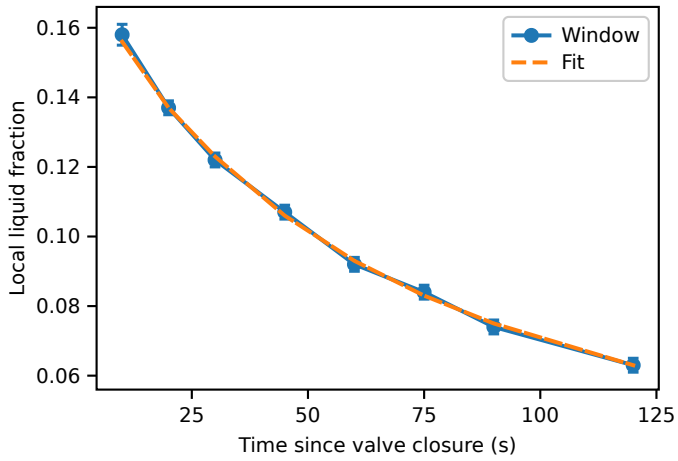


Fig. 1 | Liquid fraction at the observation window. Points give the mean across six preparation days, with one standard error. The curve is the reciprocal fit.

A simple estimate indicates when neighboring places should become confused. For a delay h between pours and a time t since the younger pour, the wetness difference on the fitted curve is

$$\delta f = \frac{f_0 \tau h}{(\tau + t)(\tau + t + h)}. \tag{2}$$

With $h = 10$ s, the last two places differ by 0.0146 at $t = 20$ s, the first two by 0.0069 at $t = 60$ s. Propagating a single-vessel error of 0.004 through the derivative of equation (1) gives age errors of 2.5 s at 20 s and 6.2 s at 70 s. The best-separated adjacent pair should thus be the last two places in the service book. In particular, the assignment of a head of delegation is a harder measurement than the assignment of its assistant. This prediction is made at fixed service interval. Cutting the interval to accommodate a larger delegation changes all the pairwise resolutions, even if the time allowed for the toast is unchanged.

Six-place rehearsals

A table occupied six positions in a straight row behind one mask. Labels on the underside of the vessels identified office rather than rank. The day’s service schedule gave each office a rank and the controller filled the positions in that order. We randomized the locations and the correspondence between office and rank anew at every rehearsal. No delegation attended. At 20 s after closing the sixth valve the shutter recorded all six windows. A blind reader received the windows as separate, randomly ordered crops from that frame. The crop identifiers were matched to offices only after the reader had returned an ordering. This prevents a gradient in backlighting or familiarity with the seating diagram from serving as an alternative registry[11,12].

We recovered the complete order in 19 of 48 independent rehearsals. The much larger count of individually correct decisions, 679 out of 720, is a consequence of distance in the order. At separations of four and five service intervals every pair was correctly placed (Table 1). For nearest neighbors there were 33 inversions in 240 decisions. Among the 48 decisions restricted to the first pair there were 11 inversions, compared with 3 for the last pair. The two inversions at distance three occurred at separate tables. In each case the office served fourth appeared older than the office served first. Sorting the same heads by height, measured in an unmasked auxiliary exposure, gave 558 correct pairs. The height reader had no access to the liquid fraction measurements.

Table 1 | Pairwise comparisons for the liquid-fraction readout. Distance is the difference of positions in the written order. Each rehearsal contributes six minus the distance to the denominator.

Distance	Pairs	Recovered	Inversions
1	240	207	33
2	192	186	6
3	144	142	2
4	96	96	0
5	48	48	0

The proportion 0.943 is a mean over tables having equal size. A confidence interval calculated with 720 independent Bernoulli trials would give separate weight to every statement about the same badly filled glass. Instead we formed the fraction correct on each table, averaged the eight tables within each preparation day and calculated a Student interval on the six day means (0.925 to 0.961). A paired comparison with height on the same days favored the window fraction by 16.8 percentage points, with a day interval of 13.2 to 20.4. Neither interval includes variability among different recipes. The binary whole-table result has considerable sampling variation. For scale, one inverted pair changes a whole rehearsal from success to failure and lowers the pairwise score by only one fifteenth[13,14].

At a table with equal initial conditions equation (2) implies that a strict readout of service order requires no watch. The reader ranked offices by decreasing apparent age. Giving the reader only b, with no conversion to f and no values for τ or f_0, left the results in Table 1 unchanged for the common monotone calibration. There is a more limited invariance to the temperature of the jug. A change that shifts every glass up by the same wetness leaves its position in the order alone, but a change of temperature while serving the table has the same sign across the row as some actual precedence relations[15]. We monitored the stock temperature continuously. The largest fall during a single service was 0.3 degrees C. Applying the measured temperature coefficient separately to each vessel changed one adjacent comparison in the complete collection.

Representation of an absent office

The service register used for the vacancy trials had six numbered lines. One interior line was marked absent. At its turn the controller ran for the usual time but diverted the foam outside the table. There were five windows in the image and no empty window: the empty seat had been physically removed before each rehearsal. The shutter delay was hidden from the reader. The controller chose the unused line equally often among the four interior ranks. A vacancy at the first or last position gives no interior gap and is observationally equivalent to an ordinary table of five at an unknown time after service. We did not score these positions. A protocol distinction with no optical consequence cannot be recovered by raising the exposure[16].

For each image we ranked the five estimated ages and inserted the missing office at the largest adjacent age difference. Sorting is sufficient because a single service pause leaves one interval of 20 s and three of 10 s. The inference can fail in two ways. A narrow actual gap may become the broadest observed gap, or a pair on either side of the vacancy may exchange places before gaps are computed. No use of the nominal 20 s shutter delay was allowed in either case. In particular no reader could insert the senior office just by checking which end of a fitted line lacked an observation. The output was a numbered place in the order. The identity or national affiliation of the absent delegate was never supplied to the assay.

On this rule 30 of 48 vacant offices took their prescribed positions (Fig. 2). The fewest correct insertions were at the senior end where the observed foam was oldest. Keeping the acquisition geometry and the nominal service list, we filled five glasses together from branches of the same head in a separate age-equalized series. Diversion of the unused branch and the dummy pause occurred on the controller log only. The reader again

received five nonempty windows and supplied a rank. The answer was correct for 11 of 48. Inserting the vacancy at a uniformly chosen interior line would give 12 in expectation. We do not infer a negative effect of equalization from a difference of one rehearsal. Its purpose is to show how little of the signal survives once service ceases to write the ages into the foam.

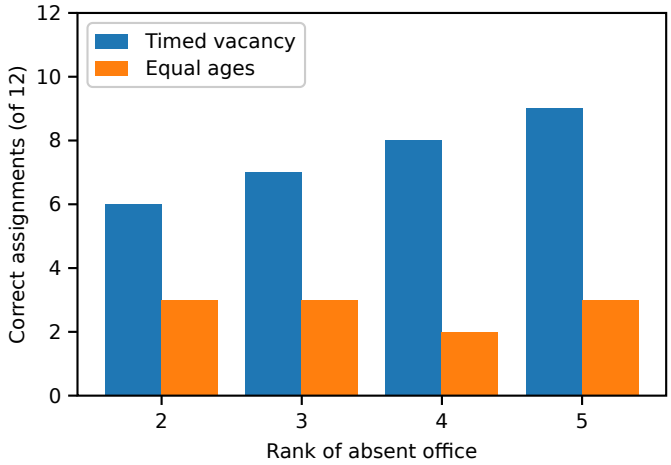


Fig. 2 | Correct assignments out of 12 per vacancy rank and condition. Positions are numbered from the earliest pour.

A useful check is to ask for the office in the absence of a pause. In a third series the controller omitted the absent delegate and immediately advanced the service clock, leaving four equal gaps between the five pours. The service book still reserved a place. The same rule inserted the omitted office correctly in 13 of 48 trials. Thus retaining an accredited member on paper is insufficient. Time must actually be spent on that member for its precedence to have an optical expression. This distinction matters at a rehearsal, where a clock operator may reasonably accelerate the exercises once a missing guest is announced. The column headed continuous service in Table 2 refers to this variant.

Table 2 | Interior vacancy assignments in balanced groups. Each set comprises six preparation days, eight independent tables per day and five columns per table.

Service	Separated pours	Reserved interval	Correct / 48
Timed vacancy	Yes	Yes	30
Equal ages	No	Log only	11
Continuous service	Yes	No	13

A rank by itself is a weak check when there are only four possibilities. We therefore inspected the gap sizes before unmasking the vacant line for the equal-age and continuous sets. For the timed-vacancy series the median observed gap at the true line was 18.7 s with an interquartile range of 12.9 to 23.6 s. In continuous service it was 9.1 s (5.1 to 13.4). These summaries refer to a gap between consecutively ranked observations at a specified rank. The ratio of the medians has no exact expectation of two, since sorting noisy ages tends to compress some spacings even at constant filling rate. On five tables the inferred age of the most senior occupied place exceeded the calibration ages near its local neighbors by more than 12 s. Large isolated age errors in the interior tend to put the vacancy on the wrong side of the same person.

Loss of resolution at late readings

In a further acquisition we retained each ordinary six-place table after its assigned frame and exposed it at two later shutter delays. This did not require a second pour. The 48 tables at each delay in Fig. 3 are therefore the same material objects, and the errors indicated are between-day standard errors at a given instant. At 60 s after the last pour the pairwise score was 639/720. At 100 s the reader recovered 584 pairs. The oldest vessel in the last condition lies outside the time range over which we calibrated the inverse function. Its rank still depends only on the sign of the wetness difference. We excluded these late readings from estimates stated in seconds. In particular, we did not estimate an apparent length of service for a delegation from a nearly dry column.

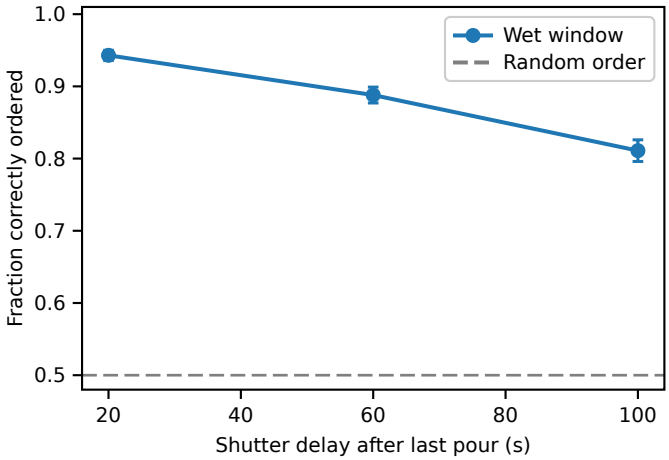


Fig. 3 | Repeated exposures of the full six-place tables. Points give proportions of correctly ordered pairs; bars show one standard error of the day means.

The loss in Fig. 3 could reflect changes in structure as well as drainage. At 20 s after the final pour the area-weighted mean bubble radius at the wall differed by 7% between the youngest and oldest groups. That difference reached 14% in the last frame. Film breakage was visible in a few windows at late times. Excluding windows whose largest cell filled over a quarter of the crop removed a disproportionate number of older glasses. We retained them. Discarding every inconvenient window while continuing to score pairs involving the omitted offices would favor the younger and less senior members of the table. The appropriate unit for a late exposure is a whole table with whatever foam remains in view[17,18].

Interpretation

The written seniority of an office has no effect on surface tension. It determines when the controller opens its valve; the sheet of instructions could equally have specified increasing shoe size. This is a useful restriction on the inference. At a second table governed by alphabetical seating, the same camera would measure alphabetical service with the same accuracy and would return an incorrect diplomatic order if the two differed. Parallel pouring produces two incompletely related clocks. An image can order members within each server’s sequence when the assignment of servers is known. In the absence of a shared start time there need be no observable relation between the oldest glass on one tray and the second oldest glass on the other. A complete written order assembled by arbitrarily interleaving the traces would introduce information that the material did not receive.

The reference surface is another practical restriction. In our vessels the same horizontal band of glass remained covered by foam throughout the first exposure. A photograph of a stemmed glass taken at an angle may show a wet tongue of liquid on the back wall behind an independent piece of foam on the front. Light from both is superimposed on the archive negative. A single grey value cannot distinguish the two contributions. On a ceremonial photograph with a usable edge view but an unknown glass composition, the strongest attainable conclusion may be that two places were served at different times. The optical calibration in A material time for the upper part of the glass should not be applied retrospectively to a polished silver cup.

What survives in a series of such photographs depends on when the material was reset. If every glass is topped up during an address, the new fronts pass the observation window at a time set by the replenishment round. The table can then return the seniority of the second round even when the first round followed a different rule. A vacancy honored only during the initial pour will have no rank in the newer layer. For the Veslan books the decisive item is consequently the notation for a refill between the opening address and the photographed toast. Neither of the two surviving diagrams records it.

Methods

Preparation and optical calibration

A single stirred bath supplied preformed aqueous foam to six matched passages. The solution was a commercial grade alkyl sulfate in water, with sodium chloride held at the same concentration throughout a preparation day. This choice gave sufficient conductivity for the calibration probe. Bubbles at discharge had a number-median radius of 0.46 mm (interquartile range 0.36 to 0.58 mm). The foam column at the end of a fill was 48 mm high and 36 mm wide across the observation faces. A submerged inlet below the head let liquid from the channels drain into a common lower compartment in each glass. No compartment communicated with another glass. The viscosity of the continuous phase at 21 degrees C was 1.08 mPa s. We used fresh solution for every day of calibration and service. The six matching vessels were washed and assigned to different positions between services. To avoid valve history as a surrogate for order, flow was maintained through bypasses between fills[19].

Within the band, a flat-field correction used an image of the same dry window in the same position. The intensity threshold for the liquid network was the crossover between the two peaks of the calibration histogram. We held it fixed within a day. On sacrificial cells containing the same foam, a short electrode pair centered at the band measured the ratio of the foam conductance to that of the liquid. A removable horizontal slice from a second matched cell was weighed immediately on collection. The conversion used 24 independent slices across the range $0.055 < f < 0.17$; the root mean squared difference between the mass estimate and the window estimate was 0.004. The band is wide enough to average over several bubble diameters but narrow enough to exclude the visible drainage front at the bottom. Substituting full-height averages introduces a different dependence on end conditions and deserves a separate calibration[20,21].

We checked shutter skew with a bar of diodes bearing the same electrical pulse at every column. The maximum difference across the width at the operating exposure was 8 ms. Compared with equation (2) this produces a wetness shift below 0.00002 at the youngest column. Valve delays were larger: the standard deviation of discharge completion about its assigned time was 0.38 s. A deliberately overfilled place could remain younger than its neighbor despite correct timing. In twelve sacrificial six-valve series, the between-place standard deviation in initial liquid fraction after adjusting for day was 0.005. Those variations enter the observed inversion rate; we did not subtract a valve-specific correction using the service order. Liquid-only blanks before and after each series checked that cleaning water had cleared the bypasses.

Error accounting

The two pulse sequences for the vacancy experiment require different controls. Equal-age service eliminates the material record of time while preserving rank labels. Continuous service retains a physical order among occupied offices but suppresses the particularly long interval due to absence. The equal-age series alone could have left open the possibility that gains on the main task simply reflected our ability to rank any five glasses. To score a vacancy the system must rank them and locate a missing interval. We kept the latter requirement even if two age estimates were equal at the display precision. Ties were broken by the original unrounded value. In case of an exact equality we had assigned independent random priorities before the experiment; none were required at instrumental precision.

The six calibration mornings were disjoint from the six service days. Each service day contained eight complete tables and eight of each vacancy condition, in random blocks of four, one table per condition. The additional frames in Loss of resolution at late readings were taken before any vessels moved. To prevent the reader recognizing a table by the shape of a bubble from its early exposure, we randomized both crop numbers and processing order across frames. The scoring program used only the wet pixel counts. The height baseline used the median head height across the unmasked central 12 mm of the auxiliary view. In the few columns with a tilted head the center height agreed better across repeated readings than the maximum height above the rim[22].

Comparison under a common unknown exposure time

For occupied ranks $r = 1, \dots, 6$ let $t_r = T + (6 - r)h$, with T the time after the last pour. On the calibration curve $u(f_r) - u(f_s) = (s - r)h$. Dropping one interior rank leaves one inter-age gap of $2h$ and the remaining three gaps of h. Both assertions hold for any T in the calibration window, which justifies the blind insertion rule in Representation of an absent office. Even if the fitted scale $f_0 \tau$ is wrong by a common positive factor, all differences in u acquire the same factor and their relative order is unchanged. An error in the shape of the calibration curve, for instance a true power of $1 + t/\tau$ different from one, need not cancel. On the 36 calibration columns the largest change of inferred spacing between a reciprocal fit and a pooled monotone spline was 2.1 s over any 10 s interval between ages 20 and 70 s.

There is no identical cancellation for absolute ages. At one table an unknown T and a uniform shift in the pouring schedule enter the data in the same way. Choosing a fixed clock origin would allow an artificial distinction between an absent first place and an absent last place. To see this directly, remove the first of six ages and shift all five remaining ages forward by h. Their set now coincides with the ages obtained by removing the last. In particular, seniority of an absent presiding officer is unidentifiable in our blind experiment even though a comparable absence in the middle of the table is recoverable. A separate record that the toast followed shutdown by exactly twenty seconds would settle that question.

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References

1. Dornat, A.V. *The Veslan Service Register* (Linden Reach Editions, 1996).
2. Celik-Soren, A.B. Reserved intervals in formal table service. *Studies in Ceremonial Records* (2013).
3. Marveil, I.S. Two revisions of the Veslan precedence diagram. *Quarterly of Manuscript Protocol* (2019).
4. Weaire, D. & Hutzler, S. *The Physics of Foams* (Clarendon Press, 1999).
5. Verbist, G., Weaire, D. & Kraynik, A.M. The foam drainage equation. *Journal of Physics: Condensed Matter* **8**, 3715–3731 (1996).
6. Neethling, S.J., Lee, H.T. & Grassia, P. The growth, drainage and breakdown of foams. *Colloids and Surfaces A* **263**, 184–196 (2005).
7. Feitosa, K., Marze, S., Saint-Jalmes, A. & Durian, D.J. Electrical conductivity of dispersions: from dry foams to dilute suspensions. *Journal of Physics: Condensed Matter* **17**, 6301–6305 (2005).
8. Mircek, I.L. Calibration of a wall-border image against a sliced column. *Interfacial Measurement Notes* (2020).
9. Koehler, S.A., Hilgenfeldt, S. & Stone, H.A. A generalized view of foam drainage: experiment and theory. *Langmuir* **16**, 6327–6341 (2000).
10. Cox, S.J. et al. Applications and generalizations of the foam drainage equation. *Proceedings of the Royal Society A* **456**, 2441–2464 (2000).
11. Parin-Luez, T.J. A mask for multiple glass columns. *Optical Cell Reports* (2014).
12. Belda Vesik, N. Randomized positions for repeat imaging of wet interfaces. *Interfacial Measurement Notes* (2018).
13. Kendall, M.G. A new measure of rank correlation. *Biometrika* **30**, 81–93 (1938).
14. Davison, A.C. & Hinkley, D.V. *Bootstrap Methods and Their Application* (Cambridge University Press, 1997).
15. Manikantan, H. & Squires, T.M. Surfactant dynamics: hidden variables controlling fluid flows. *Journal of Fluid Mechanics* **892**, P1 (2020).

16. Resan, O.Y. Unoccupied places in the service registers. *Studies in Ceremonial Records* (2005).
17. Drenckhan, W. & Hutzler, S. Structure and energy of liquid foams. *Advances in Colloid and Interface Science* **224**, 1–16 (2015).
18. Sivren, A.F. Selection of surviving cells in long optical records. *Quarterly of Wet Materials* (2015).
19. Kanat-Orbel, J. Bypass equilibration in foam dispensing arrays. *Fluid Cell Engineering* (2021).
20. Rasti, V.K. Wall angles and the sampling depth of an optical bridge. *Optical Cell Reports* (2010).
21. Saint-Jalmes, A. Physical chemistry in foam drainage and coarsening. *Soft Matter* **2**, 836–849 (2006).
22. Vayel, A.N. Reading irregular head heights on narrow foam columns. *Fluid Cell Engineering* (2017).