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Wear-induced chirality and frustrated re-pairing in domestic sock populations

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New crew socks with a reciprocated heel are mirror-symmetric, but with use the heel pocket rotates toward the geometry of the foot on which the sock is usually worn. A wearer who senses the rotation tends to put the sock back on the same foot, which reinforces it, while laundering partly releases it. We treat this as reinforcement-driven breaking of a discrete symmetry, coupled to the random re-pairing that follows each wash when a household owns M indistinguishable pairs. Random re-pairing produces same-handed, frustrated pairs with expected fraction $(M-1)/(2M-1)$, and in mean-field theory the achiral state is stable only for $M > M_c = 1/(2-g)$, where g is the product of the wearer's comfort sensitivity and the imprint efficiency of the knit. Wear-and-wash cycling on mirrored foot forms and a blinded donning experiment give $g = 1.77 \pm 0.11$. Flatbed scans of 3,814 socks from 61 households show bimodal heel-rotation distributions in small cohorts and unimodal ones in large cohorts. The curvature of the distribution at zero rotation changes sign at $M_c = 4.3^{+1.0}_{-0.7}$, consistent with the prediction. Cohorts kept paired through the wash remain chiral at every M, and wearer-resolved data show the transition moving to larger M as comfort sensitivity increases.

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I. INTRODUCTION

A circular-knitted crew sock with a reciprocated heel has a mirror plane. The heel pocket is formed by short rows worked symmetrically about the axis of the tube, and a new sock fits either foot equally well. Feet are not like this. The posterior heel sits medial to the long axis of the leg by an amount that varies between people but is mirrored between the two sides, so a sock worn on one foot is loaded asymmetrically at the heel. If any of that deformation survives the night and the next wash, a sock that has mostly been worn on the right foot will end up with a heel pocket rotated slightly toward the right-foot geometry. Of the 88 adult wearers in our household survey, 49 agreed that their older socks fit one foot better than the other. We have not found a measurement. Statistical physics has looked at household objects before, and a string tumbled in a box knots spontaneously with a probability that rises with its length [1]. Socks seem to have escaped attention.

What happens next is a problem in statistical mechanics. A wearer who can feel the rotation, even without noticing that they feel it, will tend to put the sock back on the foot it already fits, and this deepens the rotation. Reinforcement loops of this kind break a discrete symmetry spontaneously in autocatalytic chemistry [2–4] and in urn processes, of which the lock-in of competing technologies is the best-known economic case [5, 6]; Castellano et al. [7] review the social examples. Laundering pushes the other way. Wetting and agitation release part of the set in a cotton-rich knit [8, 9].

One ingredient has no obvious counterpart in those systems. Socks are worn in pairs but washed one at a time. A household that owns M identical pairs loses the original pairing in every wash and draws new pairs more or less at random from the clean pile. If both socks in a new pair have been imprinted toward the same foot, one of them has to be worn on the wrong side, and the pair is frustrated in Toulouse's sense [Toulouse [10]]: no orientation satisfies both members. In a spin glass the frustrated bonds are quenched [11, 12]. Here they are redrawn every cycle, so on average frustration weakens the reinforcing field instead of freezing the system.

The competition between reinforcement and random re-pairing sets a critical cohort size, below which socks acquire a handedness and above which they stay achiral. Section II sets out a minimal stochastic model of wear, donning and laundering, and solves it in mean-field theory. Section III checks by an order-of-magnitude estimate that the coupling is of order one. Section IV describes a wear rig, a blinded donning experiment and a household survey; Sec. V gives the results and Sec. VI their limitations.

II. MODEL

A. Imprint and relaxation

Let θ_i be the heel-pocket rotation of sock i: the angle between the bisector of the heel-pocket seams and the axis of the leg tube, with the sock laid flat (Sec. IV.A). We take $\theta > 0$ for rotation toward the right-foot geometry. A sock held on a foot indefinitely would approach

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the rotation θ_s imposed by that foot. We call θ_s the saturation angle and work with $\chi_i = \theta_i/\theta_s \in [-1, 1]$.

Each wear–wash cycle n is modelled as a linear imprint followed by partial relaxation,

$$\chi_i^{(n+1)} = (1-\lambda)\left[(1-\varepsilon)\,\chi_i^{(n)} + \varepsilon\, s_i^{(n)}\right], \qquad (1)$$

where $s_i^{(n)} = +1$ if the sock is worn on the right foot in cycle n and -1 if it is worn on the left. The imprint efficiency ε is the fraction of the remaining mismatch made permanent by one wear, and λ is the fraction of the accumulated set released by one wash. Iterating Eq. (1) with s held fixed gives a geometric approach to $\chi_\infty = \pm K$, with

$$K = \frac{(1-\lambda)\varepsilon}{1-(1-\lambda)(1-\varepsilon)}, \qquad (2)$$

and relaxation time $\tau = [1-(1-\lambda)(1-\varepsilon)]^{-1}$ cycles. Below we write $a \equiv (1-\lambda)(1-\varepsilon)$ and $b \equiv (1-\lambda)\varepsilon$.

The linear form of Eq. (1) is a simplification. Knits recover from strain in a strongly nonlinear way [8, 13], and the imprint will depend on footwear and on how far the wearer walks. We absorb all of this into ε, which we treat as a property of the sock model and measure in Sec. IV.B.

B. Donning

When a wearer puts on a pair (i, j) we assume that sock i goes on the right foot with probability

$$P(s_i = +1) = \frac{1}{2}\left[1 + \tanh\!\left(J\,\frac{\chi_i - \chi_j}{2}\right)\right], \qquad (3)$$

and sock j on the other foot. The comfort sensitivity J measures how strongly the orientation of the pair follows the difference in imprint. A wearer with $J = 0$ pays no attention, and one with $J \to \infty$ always puts the more right-handed sock on the right. Participants in the donning experiment (Sec. IV.C) were not told what was being recorded, and we read J as an involuntary preference, not a decision. For a pair with $\chi_j = -\chi_i$, Eq. (3) reduces to the familiar form $\frac{1}{2}[1 + \tanh(J\chi_i)]$.

C. Re-pairing

A cohort is a set of socks that the wearer cannot tell apart by eye: same model, same colour, same size. In a cohort of M pairs, laundering is followed by re-pairing, which we model as a uniformly random perfect matching of the $2M$ clean socks. A pair is frustrated if its two socks have imprints of the same sign.

Proposition 1 (Frustrated fraction)**.** *Let a cohort contain $2M$ socks, M with $\chi > 0$ and M with $\chi < 0$, and let the socks be paired by a uniformly random perfect matching. Then the expected fraction of frustrated pairs is*

$$f_M = \frac{M-1}{2M-1}. \qquad (4)$$

Proof. Fix one sock. Its partner is uniformly distributed over the other $2M-1$ socks, of which $M-1$ have the same sign, so the pair containing it is frustrated with probability $(M-1)/(2M-1)$. Summing over all $2M$ socks counts every pair twice. The expected number of frustrated pairs is therefore $M(M-1)/(2M-1)$, and dividing by M gives Eq. (4). ∎

So $f_1 = 0$, since a unique pair cannot be mis-paired, $f_2 = 1/3$, and $f_M \to 1/2$ for large M. A cohort with unequal numbers of the two signs has a larger frustrated fraction. For the cohorts studied here the correction is small, and we carry it as a systematic error (Table III). In a frustrated pair the difference $\chi_i - \chi_j$ is small, Eq. (3) gives roughly even odds, and we set $P = 1/2$ for such pairs.

D. Mean-field theory

Suppose that all socks in a cohort carry the same $|\chi|$ and that the pairing is redrawn every cycle. A sock with $\chi > 0$ is then in an unfrustrated pair with probability $1 - f_M$, in which case it goes on the right foot with probability $\frac{1}{2}[1 + \tanh(J\chi)]$; otherwise it is in a frustrated pair and goes on either foot with equal probability. The mean of s is $(1 - f_M)\tanh(J\chi)$, and the fixed point of Eq. (1) satisfies

$$\chi^* = K\,(1 - f_M)\tanh(J\chi^*). \qquad (5)$$

With $y = J\chi$ this is the Curie–Weiss equation $y = G_M \tanh y$ [14, 15], with effective coupling

$$G_M = g\,\frac{M}{2M-1}, \qquad g \equiv JK. \qquad (6)$$

The achiral solution $y = 0$ loses stability when $G_M > 1$. Since G_M falls monotonically from g at $M = 1$ to $g/2$ as $M \to \infty$, the outcome depends on where g sits relative to 1 and 2.

Corollary 2 (Critical cohort size)**.** *If $g \le 1$ the achiral state is stable for every M, and if $g \ge 2$ it is unstable for every M. For $1 < g < 2$ it is unstable if and only if $M < M_c$, where*

$$M_c = \frac{1}{2-g}. \qquad (7)$$

A wearer with $g \geq 2$ would polarise a cohort of any size. Everything below concerns the window $1 < g < 2$, which is where the measured parameters put a typical wearer (Sec. V.A). In this window M_c depends steeply on g, since an uncertainty δg moves it by $\delta g/(2-g)^2$. This will limit how well the laboratory measurements on their own can predict the transition.

E. Fluctuations

Each wear is a coin flip, so Eq. (1) carries noise of order b per cycle. Treating y as continuous and setting $\mathrm{Var}(s) \approx 1$, the stationary distribution of a single sock in a cohort of size M is (Appendix A)

$$P_M(y) \propto \exp\!\left[-\kappa\left(\frac{y^2}{2} - G_M \ln\cosh y\right)\right], \qquad \kappa = \frac{2(1-a)}{J^2 b^2}. \tag{8}$$

For the parameters measured in Sec. V.A, $\kappa \approx 25$. That is small enough for $\langle|\theta|\rangle$ to stay well away from zero even when $G_M < 1$, so the mean absolute rotation shows a smooth crossover and no kink. A sharper diagnostic is the curvature of the log-distribution at the origin,

$$c_M \equiv -\left.\frac{\partial^2 \ln P_M}{\partial \theta^2}\right|_{\theta=0} = A\,(1 - G_M), \qquad A = \frac{\kappa J^2}{\theta_s^2} = \frac{2(1-a)}{b^2 \theta_s^2}, \tag{9}$$

which changes sign exactly at M_c for any κ. The approximation $\mathrm{Var}(s) \approx 1$ is exact at $y = 0$, so Eq. (9) does not depend on it, and A does not depend on J. In Sec. V we use c_M, together with Hartigan's dip test for unimodality [16], as the signature of the ordered state.

III. ORDER-OF-MAGNITUDE ESTIMATE

Before describing the experiments we check whether $g = JK$ should be of order one.

Take the imprint first. Cotton recovers poorly from strain compared with polyamide or elastane [8], and the heel of a cotton-rich sock is loaded a few thousand times per wear (about 3.5×10^3 steps per foot on a day of 7×10^3 steps). Suppose a fraction r of the loop deformation imposed during the day is not recovered overnight, and a fraction q of that survives the next wash as a lasting change in loop geometry. With r between 0.1 and 0.3 and $q \sim 0.1$, $\varepsilon \sim 10^{-2}$. Dimensional-change data for hosiery under repeated laundering [9] suggest that a single wash releases well under one percent of an established set, so λ/ε is a few tenths and

$$K \approx \left(1 + \frac{\lambda}{\varepsilon}\right)^{-1}, \qquad 0.7 \lesssim K \lesssim 0.8. \tag{10}$$

Now J. From Eq. (3), the rotation θ_{75} at which an unfrustrated pair is put on the right way round three times in four satisfies $J\theta_{75}/\theta_s = \operatorname{artanh}\tfrac{1}{2} \approx 0.55$. The back of an adult heel has a radius of curvature of roughly 35 mm, so $\theta_{75} = 2^\circ$ moves the heel seam about 1.2 mm across it. We know of no direct measurement of how well people detect a displaced seam at the heel. If θ_{75} lies between 1° and 3°, and $\theta_s \approx 7^\circ$,

$$J = \frac{\theta_s}{\theta_{75}}\,\operatorname{artanh}\tfrac{1}{2}, \qquad 1.3 \lesssim J \lesssim 3.8. \tag{11}$$

The estimate puts g somewhere between 1 and 3. That is too loose to say on which side of $g = 2$ a typical wearer sits, or whether the window $1 < g < 2$ in which M_c is finite is occupied at all. It does say that the coupling is neither negligible nor overwhelming. If a finite M_c exists it should be a small integer: $g = 1.5$ gives $M_c = 2$ and $g = 1.9$ gives $M_c = 10$.

IV. EXPERIMENTS

A. Heel-rotation angle

Socks were scanned on an A4 flatbed scanner at 600 dpi (42 µm per pixel), laid flat with the heel pocket uppermost under an acrylic plate weighted to 2.1 kg, about 1.0 kPa over the area of a sock. The tube axis was taken as the midline between the two side folds, fitted over 80 mm of leg above the heel. A reciprocated heel shows as two diagonal lines of short-row joins meeting at the base of the heel. We fitted a straight line to each with a ridge detector on the green channel in Fiji [17] and defined θ as the angle between their bisector and the tube axis. Socks were scanned at least 24 h after the last wash, at 35–60% relative humidity.

The uncertainty on a single θ is dominated by how the sock lies on the platen. Table I lists the contributions, which add to 0.51°, about a fourteenth of the saturation angle. The largest systematic term is asymmetry knitted in at manufacture. We measured it on 120 unworn socks of six models, whose θ distribution is centred at 0.03° with a standard deviation of 0.26°.

TABLE I. Error budget for a single heel-rotation measurement. Statistical terms come from repeated placement; systematic terms from the comparisons listed. Contributions add in quadrature.

Source	Determined from	σ_θ (deg)
Pixel resolution	600 dpi, line fit over 60 mm	0.04
Placement on platen	40 socks, 10 placements each	0.31
Compression under plate	1.0 kg vs 2.1 kg plate	0.18
Seam detection	algorithm vs two manual tracers	0.22
Humidity, time since wash	rescans at 24 h and 72 h	0.12
Knit-in asymmetry	120 unworn socks, 6 models	0.26
Total		0.51

B. Wear rig

To measure ε and λ we built a pair of foot forms from a 3D scan of one volunteer's right foot (EU size 42) and its mirror image, cast in rigid polyurethane with a 4 mm silicone skin over heel and sole. Each form is driven into a sprung platform by a pneumatic cylinder through a heel-strike, mid-stance and toe-off sequence at 0.5 Hz, with a peak load of 820 N. One rig wear is 3,500 cycles (just under 2 h) followed by 14 h unloaded at 21 °C. This compresses a day's loading into a shorter time; we come back to whether that matters in Sec. VI.

Forty-eight new socks of one model (80% cotton, 18% polyamide, 2% elastane, reciprocated heel) were assigned at random to the two forms and taken through 120 wear–wash cycles, followed by 60 cycles of washing alone. Washes were at 40 °C in a domestic front-loading machine with a 1,200 rpm spin, followed by line drying, and each sock was scanned every tenth cycle. We obtained the saturation angle from photogrammetric reconstructions of the heel seams of 12 socks while on the forms, giving $\theta_s = 7.0 \pm 0.4^\circ$.

C. Donning experiment

Fifty-two adults from the household survey (Sec. IV.D) took part. Each was seated, blindfolded and handed 20 pairs of rig-prepared socks with imprints $\pm\theta$, four pairs at each of $\theta = 0.5^\circ, 1^\circ, 2^\circ, 3^\circ$ and 4.5°, in random order and with random orientation in the hand. They were asked to put the socks on and walk four steps, then take them off again. They were told that the study concerned sock fit. The outcome is whether the right-imprinted sock ended up on the right foot. We fitted Eq. (3) by maximum likelihood with a random effect for participant.

D. Household survey

We visited 61 households in two cities, recruited through notices in public libraries and a running club. Every sock belonging to an adult regular wearer ($n = 88$) was scanned during the visit, 3,814 socks in all. Children under 16 were excluded because foot growth changes θ_s during the life of a sock. Each wearer sorted their socks into cohorts, which we checked against the labels, and estimated the age of each cohort. From the age and the number of pairs in regular rotation we estimated the number of wear–wash cycles per sock. Wearers also described their laundry routine [18], including whether pairs are kept together through the wash with clips, safety pins, one mesh bag per pair, or by rolling the pair into a ball. We call such cohorts pair-preserving. For them $f = 0$ whatever M is.

The steady-state analysis requires an even number of socks and at least 80 estimated cycles, about 2τ. This excluded 38 cohorts with an odd number of socks (84 socks) and 118 cohorts that were too young (944 socks), leaving 487 cohorts and 2,786 socks (Table II). Most of the odd cohorts are the result of losses in the wash. We have not analysed them.

V. RESULTS

A. Model parameters

Fig. 1 shows the rig data. Socks on the right form rotate steadily toward positive θ and those on the left toward negative θ, at rates that do not differ detectably. A joint fit of Eq. (1) to the 48 trajectories, including the wash-only phase, gives $\varepsilon = 0.020 \pm 0.003$ and $\lambda = 0.006 \pm 0.002$ per cycle, hence, through Eq. (2), $K = 0.77 \pm 0.03$, $\tau = 39 \pm 5$ cycles and a plateau at $K\theta_s = 5.4^\circ$. The wash-only phase constrains λ almost by itself: over 60 washes the mean rotation falls from 5.2° to 3.6°.

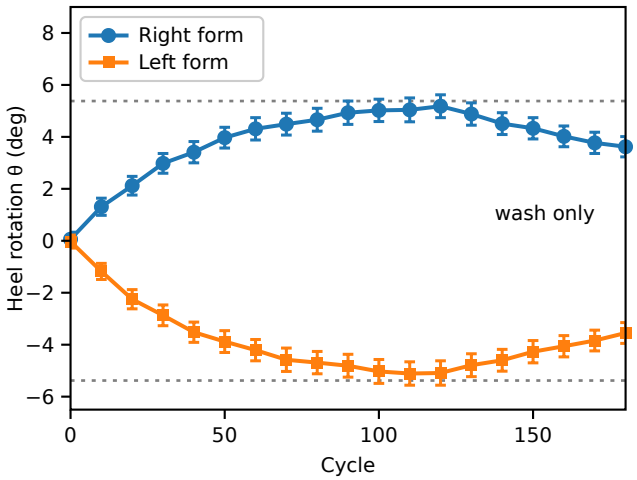


FIG. 1. Heel-pocket rotation on the right (circles) and left (squares) foot forms of the wear rig, mean and standard deviation over 24 socks each. Cycles 0 to 120 are wear and wash; cycles 130 to 180 are washes only. Dotted lines mark the predicted plateau at plus and minus 5.4 degrees.

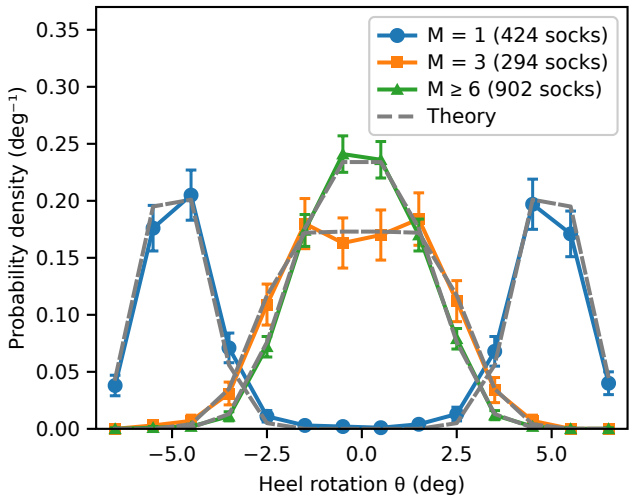


FIG. 2. Pooled distributions of heel rotation for unique pairs (circles), cohorts of three pairs (squares) and cohorts of six or more pairs (triangles). Error bars are Poisson. Dashed grey curves are the stationary distribution of Eq. (8) with parameters from the rig and donning experiments, convolved with the measurement error of Table I.

In the donning experiment the fraction of pairs put on the right way round rose from 0.57 at $\theta = 0.5^\circ$ to 0.95 at 4.5°. The fit gives a population mean $J = 2.30 \pm 0.11$, with a between-participant standard deviation of 0.39, and a threshold $\theta_{75} = 1.7^\circ$. Both fall inside the range of the estimate in Sec. III. Combining the two experiments, $g = JK = 1.77 \pm 0.11$, $\kappa = 25 \pm 9$ and $A = 2.7 \pm 0.9$ deg^{-2}. Through Eq. (7), the rig and donning data on their own predict $M_c = 4.3^{+4.0}_{-1.4}$. The interval is lopsided because M_c diverges at $g = 2$.

B. Distributions

Fig. 2 compares the pooled θ distributions of three groups of cohorts. Unique pairs ($M = 1$) are strongly bimodal, with peaks near $\pm 5^\circ$, close to the rig plateau. Cohorts of three pairs have a broad, flat top with a shallow central minimum. Cohorts with $M \geq 6$ are unimodal and centred on zero, with a width of about 1.5°, three times the measurement error. The dashed curves, described in the caption, contain no adjustable parameters.

The visible mismatch is at $M = 1$, where the measured peaks sit about 0.3° inside the predicted ones. These cohorts include socks with as few as 80 estimated cycles, and at 2τ the deterministic part of Eq. (1) is still 13% short of its plateau. Restricting to cohorts with more than 150 estimated cycles moves the peaks to within 0.1° of the prediction, at the cost of half the sample.

C. Dependence on cohort size

Table II and Fig. 3 summarise all 487 cohorts. The mean absolute rotation in Fig. 3(a) falls steeply between $M = 1$ and $M = 3$ and then levels off near 1.2°, a value set by noise rather than by order. Pair-preserving cohorts stay near 4.5° at every M. This is what one expects if frustration, rather than cohort size as such, is what suppresses the rotation.

TABLE II. Household cohorts in the steady-state analysis, by cohort size M. $\langle|\theta|\rangle$ is the mean absolute heel rotation with its standard error, c the central curvature of Eq. (9) after deconvolution of the measurement error, and p_{dip} the p value of Hartigan's dip test on the pooled cohort. PP marks pair-preserving cohorts.

M	Cohorts	Socks	$\langle\lvert\theta\rvert\rangle$ (deg)	c (deg^{-2})	p_{dip}
1	212	424	4.71 ± 0.06	−1.94 ± 0.21	$< 10^{-4}$
2	74	296	2.12 ± 0.09	−0.47 ± 0.11	$< 10^{-4}$
3	49	294	1.58 ± 0.08	−0.12 ± 0.07	0.03
4	31	248	1.41 ± 0.08	−0.02 ± 0.06	0.41
5	22	220	1.36 ± 0.09	0.08 ± 0.06	0.77
6	27	324	1.29 ± 0.07	0.10 ± 0.05	0.92
7–9	18	286	1.24 ± 0.07	0.18 ± 0.05	0.95
≥10	13	292	1.19 ± 0.07	0.20 ± 0.05	0.97
2–3, PP	14	68	4.52 ± 0.15	−1.71 ± 0.38	$< 10^{-4}$
4–6, PP	17	170	4.63 ± 0.13	−1.88 ± 0.33	$< 10^{-4}$
≥7, PP	10	164	4.48 ± 0.17	−1.62 ± 0.41	$< 10^{-4}$
All	487	2,786			

The curvature c_M in Fig. 3(b) changes sign between $M = 4$ and $M = 5$. A weighted fit of Eq. (9) to the eight bins with random re-pairing, with A and g free, gives $A = 2.58 \pm 0.24$ deg^{-2} and $g = 1.750 \pm 0.035$ ($\chi^2 = 4.1$ for 6 degrees of freedom), which puts the zero crossing at $M = 4.0$.

That number needs one correction. Cohorts near M_c relax slowly, on a time scale $\tau/|1 - G_M|$, and many of them will not have reached the stationary state. They start from $\theta \approx 0$, where the distribution is narrow and c is large and positive, so incomplete relaxation biases c upward and M_c downward. Simulating Eq. (1) with the measured distribution of cohort ages gives a shift of $+0.3$, and our result is

$$M_c = 4.3^{+1.0}_{-0.7}, \tag{12}$$

with the error budget in Table III. The fitted A agrees with the rig value of 2.7 ± 0.9 deg^{-2}, and the fitted g with $JK = 1.77 \pm 0.11$ from the rig and donning experiments. The dip test tells the same story: unimodality is rejected at $p < 10^{-4}$ for $M \le 2$ and at $p = 0.03$ for $M = 3$, and is not rejected for $M \ge 4$.

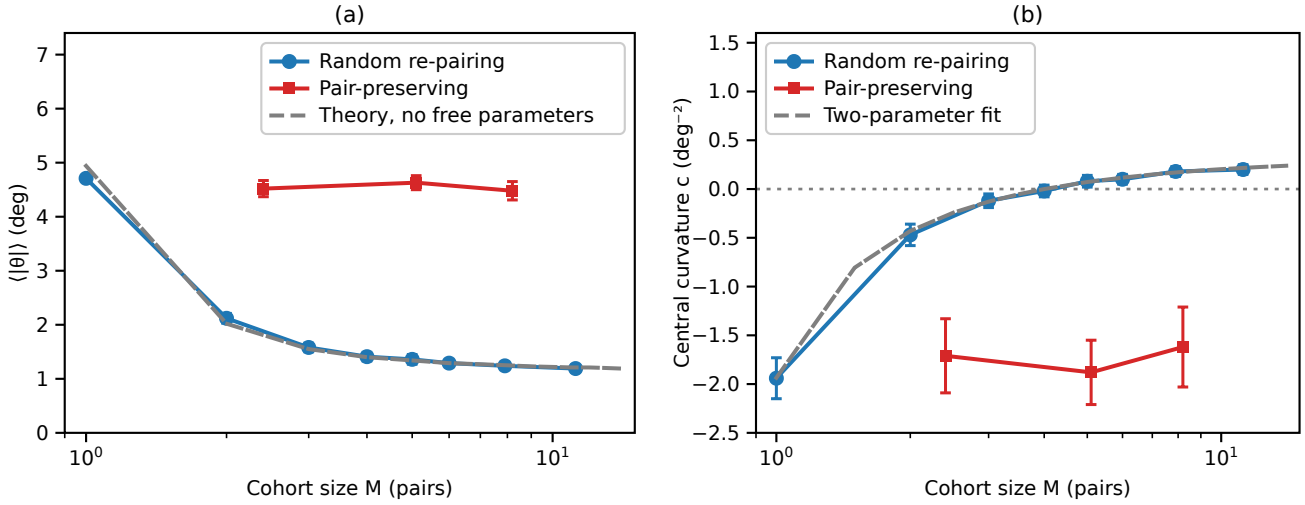


FIG. 3. Dependence on cohort size. (a) Mean absolute heel rotation. (b) Central curvature of the log-distribution, Eq. (9). Circles: cohorts re-paired at random after washing. Squares: pair-preserving cohorts, plotted at their mean M. The dashed curve in (a) is Eq. (8) with no free parameters; in (b) it is the two-parameter fit described in the text. The sign change in (b) locates M_c.

TABLE III. Error budget for the critical cohort size M_c. The finite-age correction of $+0.3$ is applied to the central value; its uncertainty is listed.

Source	δM_c
Fit of Eq. (9), statistical	$+0.9$ / -0.6
Measurement error, deconvolution	± 0.2
Finite cohort age	± 0.3
Cycle-count estimate	± 0.2
Unequal sign composition	± 0.1
Exclusion of odd cohorts	$+0.1$ / -0.0
Total	$+1.0$ / -0.7

D. Wearer dependence

Corollary 2 predicts that M_c depends on the wearer through J. For the 52 participants of the donning experiment we estimated J individually and split them into terciles, with medians $J = 1.83$, 2.31 and 2.58. With $K = 0.77$ these give $g = 1.41$, 1.78 and 1.99, and $M_c \approx 1.7$, 4.5 and 100. The last value is larger than any cohort we saw.

Fig. 4 shows c for the 301 cohorts that belong to these participants, binned by tercile and M. Unique pairs are left out, since they are ordered for every $g > 1$. The sign change moves to larger M as J increases, in the order predicted. The low tercile is already achiral at $M = 2$. The middle tercile changes sign between $M = 4$ and $M = 5$. The high tercile stays negative across the range we sampled, though it comes close to zero for $M \geq 7$. Individual cells hold between 3 and 31 cohorts and are noisy, and we have not attempted a separate fit for each tercile.

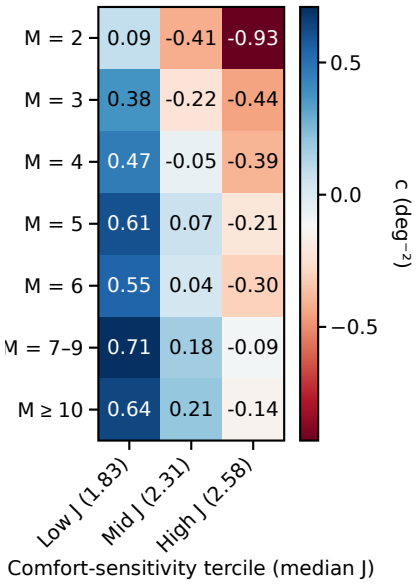


FIG. 4. Central curvature c (per square degree) of the heel-rotation distribution for the 52 wearers of the donning experiment, by tercile of individually fitted comfort sensitivity J. Positive (blue) cells are achiral, negative (red) cells chiral.

VI. DISCUSSION

The model has four parameters, and all of them were measured away from the household data, yet it reproduces the shape of the distributions in Fig. 2 and the location of the transition to within its own uncertainty. We take this as evidence that the mechanism is roughly

right, though not that the details are. The linear imprint of Eq. (1), the logistic donning rule and the assumption of a single $|\chi|$ per cohort were chosen because they are simple. A model with strain-dependent recovery would change K but, as far as we can see, not the structure of Corollary 2, which depends only on how f_M varies with M.

In Frank's model of chemical symmetry breaking, homochirality requires mutual antagonism between the two enantiomers [2]. Socks have no such antagonism directly: two right-handed socks are not in competition until a wash puts them in the same pair. The antagonism is supplied by the pairing step, and because the pairing is redrawn every cycle it averages into a weaker field, which is why the mean-field picture works as well as it does. Pair-preserving households are the quenched limit of the same model, and in that limit frustration disappears, because a fixed pair simply evolves into opposite imprints.

There are several things we could not control. The rig compresses a day of loading into two hours, and the viscoelastic recovery of cotton depends on loading time [8]; if the true ε were lower, κ would rise and the household distributions would be narrower than predicted, which we do not see. The foot forms come from one foot, while θ_s certainly varies between wearers; this broadens the $M=1$ peaks but should not move M_c, which depends on g and not on θ_s. Cohort ages are self-reported. The nine pair-preserving households chose that routine themselves and may differ in other ways. Their median wash temperature (40 °C) and the share that tumble-dry (3 of 9, against 21 of 52 in the rest) are similar to the other households, but we cannot rule out a difference in λ, and a gentler wash would raise K and favour order.

Within the model the number of identical pairs is a control parameter, and it is fixed at the point of purchase. Multipacks of five or more identical pairs are common, and they put a wearer with the median J on the achiral side of the transition. Wearers in our top tercile would need cohorts of order a hundred pairs.

Two extensions look feasible. Ambidextrous knitted work gloves have the same structure, with the imprint driven by grip rather than gait and, judging by how such gloves are sold, much larger M. In the other direction, the 38 cohorts with an odd number of socks contain an unpaired sock that is always worn with a partner from a different cohort; whether such socks drift toward the chirality their adoptive cohort lacks is a question about the coupling between cohorts, which Eq. (5) ignores.

APPENDIX A: STATIONARY DISTRIBUTION

Write $y = J\chi$. From Eq. (1), the change in y over one cycle has mean

$$\mathbb{E}[\Delta y] = -(1-a)\left[y - G_M \tanh y\right], \tag{13}$$

where we have used $\mathbb{E}[s] = (1-f_M)\tanh y$ and $JK(1-f_M) = G_M$. The variance of Δy is $J^2 b^2 \operatorname{Var}(s)$, with $\operatorname{Var}(s) = 1 - (1-f_M)^2 \tanh^2 y$. Near $y=0$ this is 1, and we use that value everywhere, which is accurate in the region that controls c_M and overestimates the width of the peaks in the ordered state.

For $b \ll 1$ the steps are small compared with the scale over which the drift varies, and the distribution of y obeys the Fokker–Planck equation [19]

$$\frac{\partial P}{\partial n} = \frac{\partial}{\partial y}\left[(1-a)\left(y - G_M \tanh y\right) P\right] + \frac{J^2 b^2}{2}\,\frac{\partial^2 P}{\partial y^2}, \tag{14}$$

with n the cycle number treated as continuous. The stationary solution with zero probability current is Eq. (8). Differentiating its logarithm twice and converting to $\theta = \theta_s y / J$ gives Eq. (9). Confidence intervals on fitted quantities were obtained by resampling cohorts with replacement [20], with 2,000 replicates.

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