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Flexural rigidity of packing inserts bearing canceled handling instructions

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Raised lettering on a packing insert uses material which can also serve as a face reinforcement. We calculate the flexural rigidity supplied by a three-line DO NOT BEND stamp under imposed cylindrical curvature. Four dies issue the instruction, append an exclamation mark, underline it, or cancel it with two diagonal strokes. All deposit the same volume on a symmetric sheet. Cancellation reduces the reinforcement measured along both print axes, yet raises its minimum over bend directions by 13.7%. The exclamation mark lowers the minimum by 1.8%. For a 0.25 mm substrate with 0.10 mm ribs, the calculated difference between issued and canceled stock is 0.056 N·mm in total cylindrical rigidity at their respective weakest orientations. Allocating 24.6% of the resin to cancellation maximizes the minimum for this block. A single-width die assigns 26.5% and comes within 0.15% of the optimum. Reducing the width of the stamp to 75% of nominal reverses the ranking.

I. INTRODUCTION

A packing insert marked DO NOT BEND has two ways to resist bending. Someone may read the instruction. If the lettering stands sufficiently far above the face of the insert, the strokes also carry axial load. The letter strokes remain under load even after the instruction has been withdrawn. This matters when the packing office cancels a stamp by drawing a cross through it. The cross adds two ribs of its own.

Printed composites permit considerable control over stiffness along a deposition path [1]. In a stretch-dominated network the directions in which that material is laid down matter even at fixed density [2]. Lettering imposes a further constraint on the path. Nearly half the length of the DO NOT BEND die studied here lies in vertical stems. The oblique strokes are short, apart from the diagonals in the two Ns. Putting more resin through the same die does little to correct that distribution.

We consider a packing insert for which the issuer specifies one mass of cured resin per stamp. A revised die can spend part of that allowance on punctuation, an underline, or cancellation. The quantity of interest is the smallest additional flexural rigidity over all directions of cylindrical bending. A shipper who knows the bending direction in advance faces a different design problem. In fact, testing the insert in its two principal printing directions gives the wrong disposition of the canceled stock.

II. STAMP GEOMETRY AND LOADING

Fig. 1 shows the four toolpaths. Letter coordinates lie on a square grid of pitch p. Each capital occupies four intervals across and six up, with one interval between letters and two between lines. The three lines share a center. The largest line is 19 intervals wide and the whole block is 22 intervals high. The exclamation mark adds five vertical intervals to the last line, in two pieces. The underline adds 19 horizontal intervals. The cancellation connects opposite corners of the original block twice. The diagonals and the letters belong to a single die. A canceled specimen therefore has narrower strokes than an issued one.

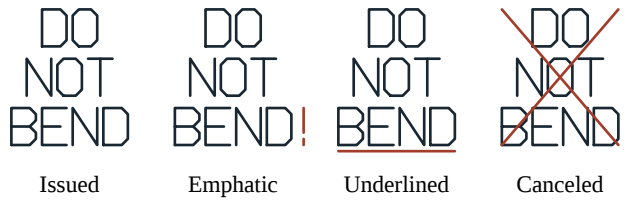


FIG. 1. Toolpaths for the four dies, at the same grid pitch. The last three differ from the issued die only along the dark red paths. Line weights in the drawing are enlarged for legibility.

We prescribe uniform cylindrical curvature κ over a sheet of area A. The direction ϕ is the direction of surface extension, measured counterclockwise from the text baseline on the upper face. The cylinder axis is perpendicular to it. The sheet has thickness h. Registered ribs on its two faces have rectangular section, height t, and axial modulus E_r. Registration here means that opposite ribs have the same in-plane coordinates viewed through the sheet. This arrangement keeps the neutral surface at mid-thickness. The base sheet is isotropic. All reported values are calculations.

A rib at angle θ_i sees axial strain $z\kappa\cos^2(\theta_i-\phi)$ at distance z from the neutral surface. We assume complete strain transfer and count only the longitudinal elastic energy of the deposited material. Integration through the rib and along each path gives the added rigidity ΔD, defined from the added energy per area $\Delta D\kappa^2/2$,

$$\Delta D(\phi) = \frac{E_r V}{A}\left[z_c^2 + \frac{t^2}{12}\right] q(\phi), \tag{1}$$

where V includes both faces and $z_c = (h+t)/2$. The dimensionless factor is

$$q(\phi) = \sum_i v_i \cos^4(\theta_i - \phi), \qquad v_i = V_i / V. \tag{2}$$

This is the longitudinal projection of the fourth-order orientation tensor [3]. Each die has a uniform stroke width, so its weights v_i are proportional to centerline length. Between dies, width varies inversely with total length. The line model assigns the crossing of two paths zero length. Widened intersections, end effects and transverse rib stress fall outside this approximation.

Only five orientation classes are needed to reproduce the issued die. In grid intervals the horizontal and vertical lengths are 44 and 80. The two 45° chamfer families each contribute $8\sqrt{2}$. The two Ns together contribute $4\sqrt{13}$ at 123.690°. On adding a mark we append its lengths, normalize the complete list, and evaluate Eq. (2) over a half-turn in 0.01° steps. The nearest grid point lies within 0.005° of a stationary minimum. Since the second derivative of $\cos^4 x$ is bounded in magnitude by 4 for angles in radians, the grid can overestimate the minimum by at most 1.6×10^{-8}. The exact angular average, $3/8$ for every die, provides a separate check on normalization.

III. DIRECTIONAL RIGIDITY

Table I gives the results. A test along the baseline assigns first place to the underlined instruction. A test perpendicular to it selects the exclamation mark. Cancellation loses to the issued die in both tests. The lowest value for the issued die, however, occurs at 34.6° , fairly close to the rising diagonal of the cancellation. Once both diagonals are present, the weak direction moves to 14.2°. The minimum rib contribution rises by 13.7%. The exclamation mark reduces it by 1.8%.

TABLE I. Equal-volume dies. Length L is per face. The subscripts on q denote the direction of extension in degrees. The last column locates the minimum over a half-turn.

Die	L/p	q_0	q_{90}	$q_{\min}$	$\phi_{\min}$
Issued	161.05	0.317	0.575	0.243	34.6°
Emphatic	166.05	0.307	0.588	0.239	33.7°
Underlined	180.05	0.389	0.514	0.258	40.9°
Canceled	219.19	0.281	0.509	0.276	14.2°

The full angular scan in Fig. 2 shows why two test strips miss the improvement. Each cancellation stroke makes an angle of 49.2° with the baseline in absolute value. The corresponding ribs have substantial extension in the oblique tests where many of the stems and bars carry little strain. The two minima either side of the baseline have different depths because both Ns run the same way. Printing the entire inscription in mirror image interchanges the two minima and leaves the lower of them unchanged.

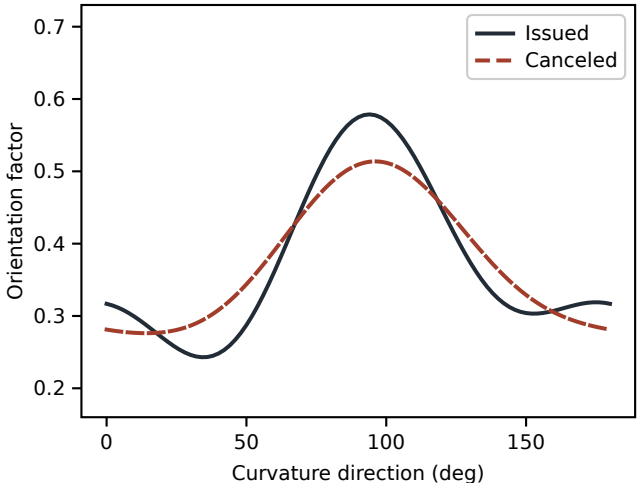


FIG. 2. Calculated rigidity of the ribs versus curvature direction at fixed deposited volume. Values are normalized by the dimensional prefactor in Eq. (1). The horizontal axis starts along the text baseline.

For scale, consider a 60×60 mm sheet with $h = 0.25$ mm, substrate modulus 2 GPa and Poisson ratio 0.30. At $p = 2$ mm, an issued die of width 0.30 mm and height 0.10 mm consumes 19.33 mm³ for both faces. The canceled die uses a width of 0.220 mm to deposit the same volume. Taking $E_r = 10$ GPa gives a prefactor of 1.69 N·mm in Eq. (1). The blank sheet has cylindrical rigidity 2.86 N·mm. Its weakest-direction rigidity becomes 3.27 N·mm with the instruction in force and 3.33 N·mm after withdrawal. The calculation puts the difference to be measured at 0.056 N·mm, about 1.7% of the issued sheet rigidity. The modulus and dimensions in this estimate are design inputs.

IV. AMOUNT OF CANCELLATION

A replacement die need not use the same width for letters and cancellation. Write f for the fraction of the resin sent to the cross. If q_I and q_X are the responses of the issued die and the bare cross, the mixed response is $(1-f)q_I + f q_X$. This construction includes a sheet on which only the cancellation remains. Stripping off the last of the lettering makes a poor insert. Its horizontal factor is just $\cos^4(49.2^\circ) = 0.183$.

There is a convenient upper bound on what any division of the resin can achieve. At $\phi_* = 17.732^\circ$ the two

pure responses intersect at 0.276608. Every mixture passes through that point. No mixture can have a minimum above it. Requiring zero angular slope there fixes the allocation,

$$f_{*} = \frac{q_{I}'(\phi_{*})}{q_{I}'(\phi_{*}) - q_{X}'(\phi_{*})} = 0.2457. \tag{3}$$

A full angular scan at this allocation gives the same value as the bound to six decimal places. An ordinary single-width die puts 26.52% of its resin into the cross and reaches 0.276192. Separate metering of the cancellation would recover only another 0.15% of the minimum rib stiffness.

The result depends appreciably on the proportions of the stamp (Fig. 3). We multiplied all horizontal coordinates, including those of the marks, by a common factor s and recalculated the lengths as well as the angles. The deposited volume and vertical pitch were held fixed. At $s = 0.75$ the diagonals spend too much of their length in a direction already supplied by the stems. The canceled die then gives 0.204 against 0.209 for the issued die. Bisection between 0.75 and 1 locates the change of order at $s = 0.7947$. An office using condensed lettering could thus obtain a small loss from the same cancellation procedure.

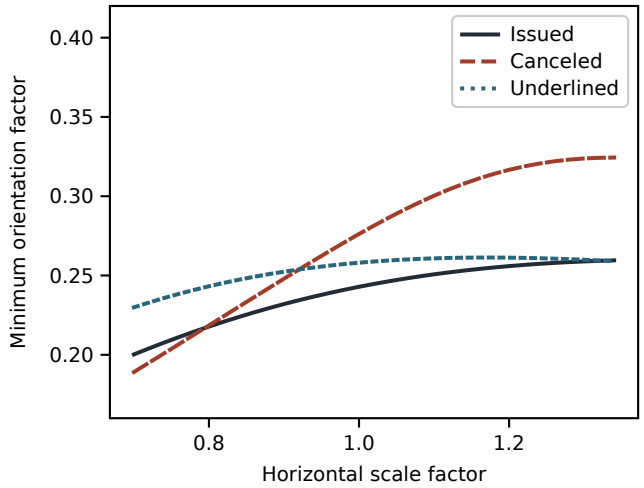


FIG. 3. Minimum orientation factor after changing the horizontal scale of the whole die. Vertical dimensions and resin volume are fixed. At unit scale the unmarked text block is 19 pitches wide and 22 high.

V. IMPLICATIONS FOR TESTING

The advantage in Table I applies to a curvature-controlled test of the marked region. On a freely bent mailer a crease can run through the blank margin and miss the stamp entirely. Even on a mandrel the estimate depends on the narrow ribs developing the surface strain. The shortest chamfers in the example are 2.83 mm long. Strain measurements at their ends would help decide whether to retain their full length in Eq. (2). The long diagonals could have a different transfer efficiency from the chamfers. A thick spot where the diagonals cross could also add mass at a different distance from the neutral surface.

A coupon comparison would require one blank for each sheet orientation, equal cured masses for the issued and canceled faces, and moment measurements during cylindrical loading near 15° and 35° as well as along the two print directions. Raising the pen pressure and drawing a cross through an existing specimen would test a different specification. At fixed line width the axial model predicts a nonnegative increase in rigidity in every direction simply because the canceling office supplies additional material.

If the issuer accepts three impressions, there is a further control that requires no special lettering. Divide the resin equally among copies at 0°, 60° and 120° on each face. The second and fourth angular harmonics of $\cos^{4}$ both sum to zero. Such a sheet has $q = 3/8$ in every direction under uniform curvature, and therefore attains the angular-average bound. The single-impression specification leaves less room for improvement. For the standard-width block studied here the strongest of the four dies for an unspecified bend direction is the one on which the handling instruction has been struck out.

[1] B. G. Compton and J. A. Lewis, Adv. Mater. **26**, 5930 (2014).
[2] G. Gurtner and M. Durand, Proc. R. Soc. A **470**, 20130611 (2014).
[3] S. G. Advani and C. L. Tucker, J. Rheol. **31**, 751 (1987).