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Holonomy and Competing Schedules in a Closed Chain of Seminar Consultations

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Pairwise requests to move a seminar time need not be jointly satisfiable when the last proposal returns to the first participant. We formulate a closed chain of consultations as a compact rotor model coupled to a fixed $U(1)$ connection. The product of link transports is invariant under independent changes of clock convention and determines whether all requests can be met. A character expansion gives the exact finite-size partition function and the response to the net requested shift. At half a scheduling period two distinct residual-mismatch configurations have equal action; their contributions round the stiff-limit cusp. For a six-person ring with a four-hour net shift in an eight-hour period, the residual is 40 minutes per link in either low-action configuration. At a dimensionless preference stiffness of six, the relative partition function is 0.0241. The calculation separates a global inconsistency of the requests from the choice of local calendar conventions.

I. INTRODUCTION

A seminar time is often fixed by a sequence of pairwise consultations. A proposed time is passed from one participant to the next, each of whom may ask for a displacement relative to the proposal they received. When the final participant returns a proposal to the organizer, the resulting time need not be the one with which the consultation began. The discrepancy can persist even if every request is individually feasible and every calendar displays the correct local time. In particular, asking everyone to convert their availability to the organizer's time zone does not change it.

We study this obstruction on a closed chain of consultations. Each participant supplies a preferred relative displacement, and the chain is periodic in the time of day. The product of the corresponding parallel transports is a compact $U(1)$ holonomy. Holonomies distinguish globally inequivalent configurations even when a connection can be removed locally, as in lattice gauge theory and the Aharonov–Bohm problem [1, 2]. Our link variables are fixed scheduling requests, rather than dynamical gauge fields; the problem is a frustrated rotor ring in a background connection. One-dimensional rotor systems admit a character expansion in which the remaining flux label is conserved around the ring [3].

There are two reasons to retain the compact model. First, a request to move a seminar from 16:00 to 09:00 on the next equivalent working day is a small displacement on a periodic schedule, not a large negative displacement on an infinite line. Second, different windings of the same consultation can compete. In a ring whose net requested shift is half a working day, there are two equally good ways to distribute the disagreement. Neither is selected by a change of time zone.

We derive the exact partition function and its response to the net requested shift. A six-person example gives a concrete scale for the effect. The calculation is an equilibrium model of stated preferences, not a model of the time taken to reach agreement; it does not require observations of actual seminars.

II. CONSULTATION AS A COMPACT GAUGE SYSTEM

Let $N \geq 3$ participants be labelled cyclically, and let T be the period after which a time slot is treated as equivalent. Participant i represents a proposed slot by an angle $\theta_i \in [0, 2\pi)$, with $\theta_{N+1} = \theta_1$. A directed request on link i specifies a preferred shift d_i from participant i's proposal to participant $i+1$'s proposal. We set $a_i = 2\pi d_i / T$ modulo 2π. The gauge-invariant mismatch is

$$\delta_i = \theta_{i+1} - \theta_i - a_i \pmod{2\pi}. \tag{1}$$

A participant's clock convention can be changed independently. Under $\theta_i \mapsto \theta_i + \lambda_i$ the same requests are represented by $a_i \mapsto a_i + \lambda_{i+1} - \lambda_i$. Each δ_i is unchanged. The convention change must also be applied to the link labels; changing the θ_i alone would change the proposals themselves. The net request and its Wilson loop are

$$\Phi = \sum_{i=1}^{N} a_i \pmod{2\pi}, \qquad W = e^{i\Phi}. \tag{2}$$

All link mismatches can vanish if and only if $W = 1$. Necessity follows by summing Eq. (1) around the ring. For sufficiency, choose any θ_1 and set $\theta_{i+1} = \theta_i + a_i$ successively. The last link then closes precisely when the sum of the a_i is a multiple of 2π.

For $W \neq 1$ a chair may make any one request appear to have zero displacement by choosing the two adjacent clock origins. In fact, one can set the first $N-1$ links to zero, leaving the entire holonomy on the last link. This is a convenient representation of the same problem, not a prescription that the last participant should

bear the whole discrepancy. An open chain has no such invariant: the same recursion removes every link variable. Closing the chain adds one independent constraint.

We assign a dimensionless action to a set of proposals,

$$S[\theta;a]=K\sum_{i=1}^{N}(1-\cos\delta_i), \tag{3}$$

with $K>0$. The cosine makes the cost periodic and symmetric under reversal of a mismatch. Near agreement the cost is $K\delta_i^2/2$, so $K^{-1/2}$ sets the angular width of a single request in the harmonic approximation. We use a common K to keep the consequences of closure separate from differences in participants' flexibility. The normalized measure defines

$$Z_N(\Phi;K)=\int\prod_{i=1}^{N}\frac{d\theta_i}{2\pi}\,e^{-S[\theta;a]}. \tag{4}$$

The integral is unchanged by a simultaneous shift of all θ_i. Fixing one proposal to zero removes that global mode and gives the same Z_N with the normalized measure. The corresponding Jacobian is constant. It should not be confused with the field-dependent Faddeev–Popov determinant of a non-Abelian gauge theory [4]. Nor do the two competing winding saddles below constitute Gribov copies: they have different mismatch configurations, not merely different descriptions of the same configuration [5].

III. EXACT PARTITION FUNCTION

Expanding each link weight in $U(1)$ characters,

$$e^{K\cos\delta}=\sum_{m\in\mathbb{Z}}I_m(K)e^{im\delta}, \tag{5}$$

where I_m is the modified Bessel function, the angular integral at each vertex enforces equality of the two adjacent integer labels. A single integer m therefore circulates around the ring, and

$$Z_N(\Phi;K)=e^{-NK}\sum_{m\in\mathbb{Z}}I_m(K)^N e^{-im\Phi}. \tag{6}$$

The expression is real and even in Φ. For fixed positive K, $I_m(K)$ decreases sufficiently rapidly with $|m|$ for the sum and its derivatives to converge absolutely. Although individual terms can be negative, the original integral is strictly positive.

We report the relative weight $R_N(\Phi;K)=Z_N(\Phi;K)/Z_N(0;K)$ and the response $J_N=-\partial_\Phi\log Z_N$. The ratio compares two fixed collections of requests at the same N and K; it is not the probability that a meeting occurs. The response measures the change in free energy with the oriented net request. With $r_m=[I_m(K)/I_0(K)]^N$, the quantities used below are

$$R_N=\frac{1+2\sum_{m\ge1}r_m\cos m\Phi}{1+2\sum_{m\ge1}r_m}, \tag{7}$$

$$J_N=\frac{2\sum_{m\ge1}m r_m\sin m\Phi}{1+2\sum_{m\ge1}r_m\cos m\Phi}. \tag{8}$$

In the gauge where $a_N=\Phi$ and all other links vanish, $J_N=-K\langle\sin\delta_N\rangle$. Vertex integration by parts gives the same expectation on every link. A local change in one preferred displacement therefore probes the same circulating response wherever the link is placed.

For small K, $r_1=(K/2)^N+O(K^{N+2})$, and the leading flux dependence is

$$R_N=1-2(K/2)^N(1-\cos\Phi)+O(K^{N+2}). \tag{9}$$

The dependence on a closed chain first appears at order N. Adding participants at fixed K suppresses that dependence: for any fixed $m\neq0$, $I_m(K)/I_0(K)<1$. This limit concerns the relative weight of the closure constraint, not the number of messages needed to organize a larger seminar.

IV. COMPETING SCHEDULES AT HALF A PERIOD

At large K, stationary configurations satisfy $\sin\delta_i$ equal on all links. For $|\Phi|<\pi$, the uniform low-action branch has $\delta_i=-\Phi/N$, with all angles taken in their principal ranges. Its action is

$$S_0(\Phi)=NK\,[1-\cos(\Phi/N)]. \tag{10}$$

At $\Phi=\pi$ the branch with $\delta_i=-\pi/N$ and the branch with $\delta_i=+\pi/N$ have equal action. These are distinct distributions of the residual mismatch. Their equality follows from reflection and periodicity, and does not require a particular choice of clock origins. For $N\ge3$ both have positive quadratic fluctuations after the common mode is removed.

The exact response must vanish at $\Phi=\pi$ by periodicity and reflection. Nevertheless, the two branches carry opposite nonzero responses in the stiff limit. Writing $\epsilon=\pi-\Phi$, their actions differ by $2K\sin(\pi/N)\epsilon+O(\epsilon^2)$. In the window $\epsilon=O(K^{-1})$, keeping both saddles gives

$$\begin{aligned}J_N(\pi-\epsilon;K)\simeq{}&K\sin(\pi/N)\\&\times\tanh[K\sin(\pi/N)\epsilon]\,.\end{aligned} \tag{11}$$

The rounding width is of order $[K\sin(\pi/N)]^{-1}$ in phase. At any finite K the exact result is smooth; a cusp arises only after taking the stiff limit. A single-branch approximation misses the cancellation at half a period.

V. A SIX-PERSON CONSULTATION

Take $T = 8$ h and six participants. Four successive requests each move the proposed time one hour later; the other two request no change. The net displacement is four hours, so $\Phi = \pi$. In either low-action branch, every link retains a 40 min mismatch relative to its preferred displacement. One branch distributes the residual in the earlier direction, the other in the later direction. Each pair of participants can satisfy its own request in isolation, but all six requests cannot be satisfied at once.

For an order-of-magnitude comparison, suppose a single request has a harmonic width of 30 min. The corresponding $K \simeq [T/(2\pi \times 0.5\,\mathrm{h})]^2 \simeq 6.5$. We use $K = 6$ as a round benchmark, for which the width is 31 min. This is an illustrative input, not a fitted parameter. At $N = 6$, Eq. (7) gives $R_6(\pi; 6) = 0.0241$: the integrated weight of proposals under the four-hour net displacement is about 2.4% of that for zero net displacement, at the same K. The quantity is a ratio of partition functions, and does not say that 2.4% of invited people attend.

Fig. 1 shows the continuous dependence on the net request for several stiffnesses.

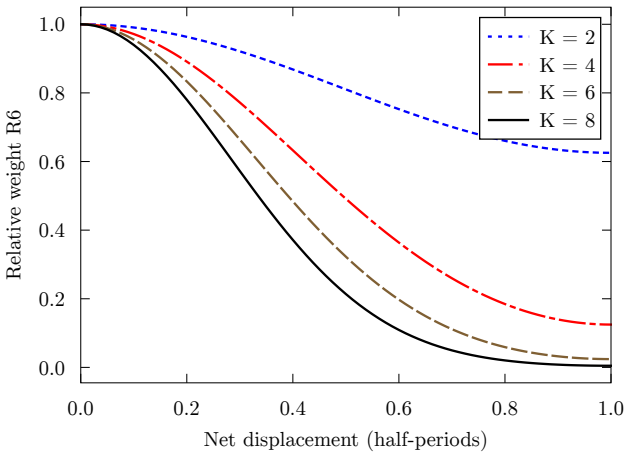


FIG. 1. Relative partition function R_6 against total requested displacement Φ/π. Curves are the exact character sum at the indicated stiffness K. A displacement of one on the horizontal axis corresponds to four hours for an eight-hour period.

Fig. 2 shows the response on the same interval. The response becomes large near half a period but returns to zero at the endpoint, where the two residual distributions contribute equally. The table gives exact values at selected points and compares different ring sizes. The increased relative weight at larger N reflects the greater number of links over which a fixed total mismatch can be distributed.

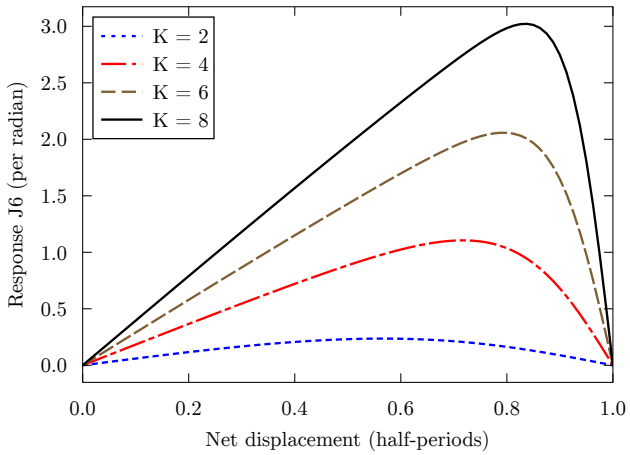


FIG. 2. Response $J_6 = -\partial_\Phi \log Z_6$ for the same parameters as Fig. 1. The exact response vanishes at half a period, even though the two stiff-limit branches have opposite nonzero slopes there.

TABLE I. Exact relative weights and response for selected rings. The response is dimensionless per radian.

N	K	$R_N(\pi/2)$	$R_N(\pi)$	$J_N(\pi/2)$
6	2	0.8102	0.6254	0.2312
6	4	0.4914	0.1248	0.8848
6	6	0.3226	0.0241	1.4285
6	8	0.2137	0.0048	1.9509
3	6	0.1045	0.0006	2.7924

The $N = 3$ row is included to show the finite-size dependence, not as an independent data set. For an open chain with $N - 1$ links, integration over successive relative angles gives $Z_{\mathrm{open}} = [e^{-K} I_0(K)]^{N-1}$, independent of all requests. Deleting a link removes a constraint and changes the model, so this partition function should not be compared numerically with the closed-ring Z_N as though the two systems had the same action.

Consider now a preliminary meeting to decide when to hold the seminar. If its consultation has the same link requests as the seminar and the two sets of proposals are independent, their joint relative weight is R_N^2. At the benchmark above this is 5.81×10^{-4}. Adding m independent preliminary meetings with the same requests gives R_N^{m+1}. This multiplication assumes that the preferences and the period stay fixed at each level. A preliminary meeting whose participants revise a request changes the holonomy and falls outside that calculation.

VI. NUMERICAL CHECKS AND SCOPE

The curves and table are evaluations of Eq. (7) and Eq. (8), not simulated seminar records. We computed I_m from its positive power series and retained $|m| \le 30$. For $N = 6$, $K = 12$ and $\Phi = \pi$, a more demanding cancellation than in the plotted range, truncation at $|m| \le 8$ gives $R = 0.00018868946$; cutoffs 12 and 30 both give 0.00018868844 to the displayed digits. For an independent check, we fixed one angle in the $N = 3$, $K = 6$ integral and used a uniform periodic grid for the other two. Grids of 32^2 and 128^2 points both give $R_3(\pi; 6) = 0.00056083174$, in agreement with the character sum at the shown precision. These checks bound numerical resolution in the examples; they do not estimate uncertainty in the model of preferences.

The dominant modeling choices are the equivalence of slots separated by T, the common stiffness and the absence of a preferred absolute time. Real calendars have holidays, room constraints and participants whose availability is not periodic. An absolute-time preference would couple to θ_i and must be transformed along with the local clock convention; it cannot generally be removed by moving all proposals together. Unequal link stiffnesses leave the holonomy condition intact but change the distribution of residual mismatches. No inference about an actual department follows without specifying those inputs.

The net shift is nevertheless an observable property of the stated pairwise requests. When it is nonzero, a request for everyone to restate the same availability in a common format can remove the local offsets from the record but cannot close the cycle. The remaining question is which request to change.

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