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Nonlocal control of depletion in a two-station allocation model

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We study depletion at an occupied serving station coupled by recorded announcements to a station with zero stock. A call understood to apply to the occupied station renews a finite response interval during which diners defer claiming its remaining food. For periodic announcements we derive the depletion time as a function of the phase offset and the probability that the remote call is excluded from the local offer record. At the reference settings, local announcements every 10 s give a mean depletion time of 18.94 s. A remote call at the midpoint of each period prevents depletion, while excluding 5% of those calls gives a mean of 421 s. With a common playback stream to separate rooms, the probability that all rooms retain their food remains positive as the number of rooms tends to infinity at fixed observation time. The common stream changes the joint distribution of the surviving stock while leaving each room's marginal depletion law unchanged.

I. INTRODUCTION

At a buffet, the station that issues an offer and the station from which food is taken can differ. A call from behind a partition may be heard at several tables. When it specifies neither a recipient nor a location, diners can interpret it as applying to the food in front of them. A service log assigns the call to its originating station. A diner waiting for somebody else to answer assigns it to the plate.

Offers have a sequence in conversation [1]. At the end of a meal, an offer can be renewed even after an earlier response has been heard [2]. We consider repeated public offers to a group in which diners defer reaching for the remaining food until the others have had time to answer. An offer from any station that is understood to apply locally renews this interval. Station labels provide a way to change that interpretation while keeping the number and timing of calls fixed.

We use a first-passage model [3, 4] to study the limiting case of two stations with no path for stock or persons to move between them. One station has the last slice of pizza and the other is empty. An announcement channel between them remains open after the second station has exhausted its stock. We calculate the conditions under which the channel prevents depletion at the occupied station. We then consider a broadcast from the empty station to several rooms with separate stocks and diners. Common resetting can correlate otherwise independent processes [5]. Here the relevant observable is the joint persistence of units of food in different rooms.

II. MODEL

Station A starts with one slice. Station B starts empty and stays empty. Four eligible diners are present at A. A local offer ends at $t = 0$, and further local offers end at integer multiples of a period P. Station B supplies one recorded offer per period, at a phase offset ϕ after the local offer. Both streams use the wording "Please take what remains." The recording at B is played according to the schedule regardless of its stock. All times refer to the ends of the calls. We omit duration effects and allow coincident calls as the limit of adjacent calls.

An offer that applies to A opens a common response interval of duration d. A diner can claim the slice after the interval has elapsed. Eligible claims have independent exponential clocks with total rate a across the diners. Any subsequent applicable offer restarts the interval. The first claim removes the slice, and we stop following that station. All claim clocks are independent of the offers.

We let q denote the probability that B's call in a given period is excluded from A's offer record. For example, a station-specific preface can exclude it while leaving it audible. Exclusions are independent between periods. The cases $q = 0$ and $q = 1$ represent an entirely shared offer record and a fully separate one. The classification of a particular call is common to all diners at A. The physical state at B does not enter this rule.

For a gap g between applicable offers, the time during which a claim can occur is $w(g) = (g - d)_+$, where $(x)_+ = \max(0, x)$. The gap survival probability is $e^{-aw(g)}$. The period at A has one gap of length P if the remote call is excluded and two gaps of lengths ϕ and $P - \phi$ if it applies. The two conditional probabilities of survival are

$$S_0 = e^{-aw(P)}, \qquad S_1 = e^{-a[w(\phi)+w(P-\phi)]}. \tag{1}$$

The integrated survival within a gap, conditional on reaching its beginning, is

$$b(g) = \min(g, d) + \frac{1 - e^{-aw(g)}}{a}. \tag{2}$$

The integrals over one period are $B_0 = b(P)$ and $B_1 = b(\phi) + e^{-aw(\phi)}b(P-\phi)$. Each local offer initiates an independent period given survival to that point. Summing the period contributions gives the mean from the first local offer,

$$\langle T \rangle = \frac{qB_0 + (1-q)B_1}{1 - qS_0 - (1-q)S_1}. \tag{3}$$

The denominator is the chance of depletion in any period that begins with the slice present.

When $P/2 \le d < P$ and $q = 0$, there is a range $P - d \le \phi \le d$ in which both gaps lie within the response interval. Then $S_1 = 1$ and the occupied station cannot deplete. With B excluded, $w(P) > 0$ and the slice is eventually taken. Opening the shared channel makes the absorbing state inaccessible.

III. NUMERICAL PROTOCOL

We set $P = 10\,\mathrm{s}$, $d = 6\,\mathrm{s}$, and $a = 0.16\,\mathrm{s}^{-1}$. This makes the mean time to first claim after the response interval 6.25 s. The response interval is a design parameter, long compared with usual gaps between conversational turns [6]. It represents the time people leave for another person to volunteer before anyone takes possession.

To check Eq. (3), we simulated 20,000 independent runs per condition. In each period we drew B's inclusion status, then drew an exponential claim delay in each gap and added it to d. A claim was accepted if its time fell inside the gap. We followed finite-mean runs to depletion. When both gaps were shorter than d, follow-up stopped at 120 s. A sweep at $\phi = 5$ s used an additional 20,000 runs at each of seven values of q. These runs were independent of the main conditions. The seed for the complete set of simulations was 340819. Intervals for means are normal Monte Carlo intervals calculated from the sample variance.

IV. RESULTS

A. Phase of the remote offer

For $q = 0$, the duration of the arrest interval in phase is $2d - P = 2$ s. At $\phi = 2$ s, A still has a gap of 8 s after the remote offer and can deplete during the last 2 s of that gap. At $\phi = 3$ s, the exposure drops to 1 s per period. The mean grows from 35.46 to 67.12 s over this shift of one second (Table I). At $\phi = 5$ s neither gap admits a claim. All three schedules deliver two announcements per period, of which one originates at a station that holds no food.

Fig. 1 shows that the arrest interval becomes a plateau at finite q. Each point on the plateau has the same mean because only periods in which B's call is excluded give A a chance to deplete. Outside the plateau some of the included calls leave time for claims, and these periods also contribute to depletion. At the edges, $\phi = 0$ or P, the remote call coincides with a local one. It changes neither the gaps nor the mean.

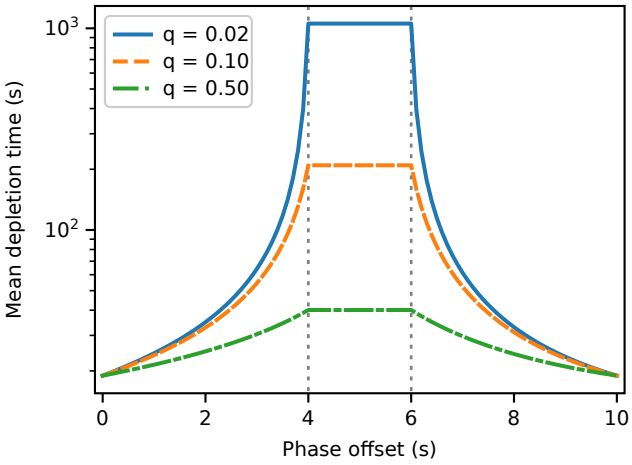


FIG. 1. Exact mean depletion time versus the phase of the remote offer for three exclusion probabilities. The vertical lines delimit the phase interval in which each included remote offer prevents depletion for the whole period.

TABLE I. Depletion times after the initial local offer. Means are from complete runs and intervals are 95% Monte Carlo intervals. The arrested condition was followed for 120 s. Counts are given out of 20,000 trials.

Remote condition	Phase (s)	q	Exact mean (s)	Simulated mean (s)	Depleted by 120 s
Excluded	5	1	18.94	$\mathbf{19.10 \pm 0.22}$	19,989
Shared	2	0	35.46	$\mathbf{35.38 \pm 0.43}$	19,550
Shared	3	0	67.12	$\mathbf{67.10 \pm 0.87}$	17,056
Shared	5	0	∞	Censored	0
Partly excluded	5	0.05	420.88	$\mathbf{421.61 \pm 5.79}$	4,995

B. Exclusion of remote offers

At the center of the plateau, $S_1 = 1$ and $B_1 = P$. During a period in which the remote call is excluded, the removal probability is $p_0 = 1 - e^{-a(P-d)} = 0.473$. The mean on the plateau can be written

$$\langle T \rangle = \frac{P}{q p_0} + \frac{B_0 - P}{p_0}. \tag{4}$$

The first term diverges as q falls. The second accounts for removals before the end of the final period. At $q = 0.05$, the mean is 421 s and about one-quarter of the runs have depleted by 120 s. At $q = 0.01$ the mean is 2,113 s. Fig. 2 shows the scaled mean $q\langle T \rangle$, which approaches $P/p_0 = 21.15$ s.

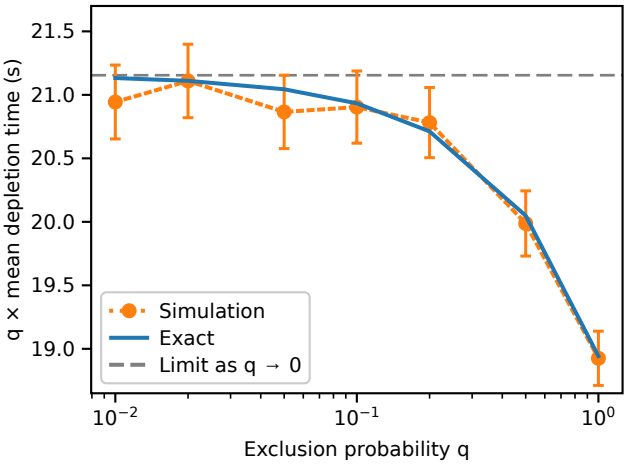


FIG. 2. Scaled mean depletion time at a 5 s phase offset. The markers are simulations with pointwise 95% Monte Carlo intervals. The dashed horizontal line shows the limit from Eq. (4).

C. Shared playback in separate rooms

We now let N occupied rooms receive announcements from the same empty station. Each room begins with one slice and its own diners. Local offers are synchronized, and the remote announcement sits at the midpoint between successive local ones. We compare a common exclusion stream, in which the same remote calls are left out in every room, to separate streams with the same per-room exclusion probability q. An excluded call exposes all rooms in the common case. Claims remain independent conditional on the complete playback record.

Let $s = S_0$ and let $C_N(k)$ be the probability that all N rooms retain their slice after k periods. With common playback each period involves either a fully protected set of rooms, with probability $1 - q$, or a period in which all N units independently survive with probability s^N. Thus

$$C_N(k) = [1 - q + q s^N]^k. \tag{5}$$

Separate streams give $C_1(k)^N$. The correlation between the survival indicators in a pair of occupied rooms is

$$\rho(k) = \frac{C_2(k) - C_1(k)^2}{C_1(k)[1 - C_1(k)]}. \tag{6}$$

The covariance arises from the common opportunity to claim. It does not require either group to hear the other group's claims.

We simulated 50,000 independent pairs of rooms per condition at $q = 0.10$, using Bernoulli outcomes for each exposed period and separate claim draws for the two rooms. At 300 s the paired survival correlation was 0.284 ± 0.010 under common playback and -0.001 ± 0.009 under separate playback, in agreement with Eq. (6) and the independent prediction of zero. Fig. 3 shows the correlations at each observation time. Intervals use the delta method applied to the four joint counts of the two survival indicators.

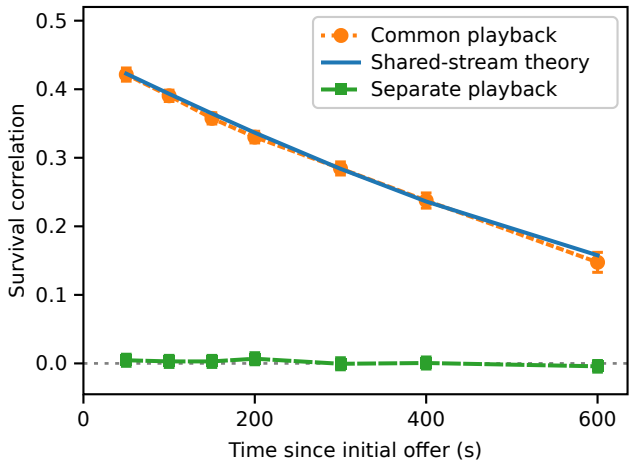


FIG. 3. Correlation of stock survival in two occupied rooms receiving the same or separate playback streams. Each condition uses 50,000 independent pairs. Bars show pointwise 95% Monte Carlo intervals.

The difference increases with the number of rooms when the outcome is retention of all the slices (Table II). For common playback the limit as $N \to \infty$ at fixed k is $(1 - q)^k$, the probability that the empty station supplies every scheduled announcement. For separate playback the all-room probability tends to zero. In the reference case, after two minutes the probability that every one of twenty slices remains is 0.282 under common playback and 9.0×10^{-6} under separate

playback. The inventories and the marginal probability of depletion in each room are the same in both cases.

TABLE II. Exact probability of retaining every slice for 120 s with $q = 0.10$. When only one room is occupied the two playback protocols are equivalent.

Occupied rooms	Common	Separate
1	0.5593	0.5593
2	0.4069	0.3128
4	0.3130	0.09784
8	0.2847	0.009574
20	0.2824	8.97×10^{-6}

V. DISCUSSION

The predicted increase in lifetime depends on whether diners understand a remote offer as applying to their plate. A test could display both station labels and keep the slice visible throughout, while routing the same recordings to the occupied room under both conditions. In one condition the remote announcement would identify its source. In the other it would omit the location. Reports of which station a call referred to would estimate q separately from the depletion time. The model places every removal on the plateau in a period with an excluded call. Measuring time since the last applicable offer would distinguish this mechanism from a general reluctance to take the slice in a room with more announcements.

The window assumption can be relaxed. If the claim hazard before d is a constant $a_0 > 0$, a fully shared centered schedule has mean $1/a_0$ even when every call arrives. An early hazard of $0.01a$ would put that mean at 625 s for the reference parameters. With both mechanisms present, claims below age d report early acceptance, whereas claims above it can occur during an excluded period. Real rooms also differ in how listeners attribute a call. Assigning inclusion to individual diners would change the all-room limit because a single listener may hear a gap that the others do not.

A station’s stock count and the service state of its announcement channel can therefore enter the depletion calculation separately. Closing the food counter at B has no effect if its recording continues. Closing the channel restores the 18.94 s local mean without moving a plate, changing the food, or admitting another diner. For installations in which several rooms share the same feed, deactivating the empty station is a coupled intervention even when each room reports its stock separately.

DATA AVAILABILITY

All data are generated from the model. The numerical protocol gives the sample sizes, parameters, and pseudorandom seed.

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