capstract:2611.00061v1 [astro-ph.CO] 20 Nov 2026

Causal reach of sequential exchanges in an accelerating universe

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(Dated: November 20, 2026)

We calculate the comoving range of exchanges in which emission on a given leg is contingent on reception of the preceding leg. For radial exchanges between two comoving stations in flat ΛCDM, the one-leg event horizon overstates the range of an m-leg null exchange by a factor m. With $H_0 = 67.4\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$ and $\Omega_\mathrm{m} = 0.315$, the first three limiting separations are 5.114, 2.557, and 1.705 Gpc, mapped to present-epoch redshifts of 1.869, 0.691, and 0.428. A fixed-frequency return signal constrained to arrive at no less than half its emission frequency reduces the two-leg range to 1.690 Gpc, or 29% of the unfiltered comoving volume. For mixed timelike and null transport we find an exact degeneracy in de Sitter space: a geodesic first leg followed by a null leg has the same reach as two legs maintained at the initial timelike speed. The degeneracy is lifted in the presence of matter. We use these bounds to distinguish incomplete exchanges from completed exchanges whose outcomes are inaccessible to the initiating station.

I. INTRODUCTION

An object can be observed on our past light cone after it has passed beyond the reach of a signal emitted here today. This distinction is usually described in terms of the particle and event horizons [1]. In an eternally accelerating universe a null ray emitted today can traverse only a finite comoving distance [2]. The standard event-horizon calculation asks about the last signal that one worldline can send to another. It takes no account of whether the recipient can act on that signal and send a new one in reply.

Here we consider exchanges whose timing is set by the arrival of preceding legs. This matters when the initiating station needs a record of an event at the remote endpoint. A successful first crossing is necessary for such a record and can occur even when a second crossing is impossible. A separate issue is whether an otherwise causally accessible reply arrives within the receiver's passband. The redshift relevant to that calculation is accumulated *after* the first crossing. The redshift used to locate the remote station, on the other hand, is measured today.

We treat the stations as comoving in a homogeneous, spatially flat background. We first calculate the reachable domains with prescribed local speeds. We then allow the initial carrier to coast, or impose a frequency cut on a returning null signal. In Sec. IV we identify which stages of a delivery record can be verified at the station that opened it.

II. PROPAGATION MODEL

A. Sequential exchanges

Let A and B occupy comoving worldlines at radial separation χ. Station A initiates the exchange at cosmic time t_0 and the endpoints alternate thereafter. A response carries an identifier specific to the preceding arrival. We require the response to be generated at its endpoint after that arrival. A prearranged transmission does not contain evidence of the arrival, even if the stations know the same schedule [3]. We assume instantaneous processing except when considering a finite turnaround allowance.

For radial propagation write the flat FLRW metric as $ds^2 = -c^2dt^2 + a^2(t)d\chi^2$, with $a(t_0) = 1$. We define the conformal distance available in proper time T on a comoving clock by

$$R(T) = c\int_{t_0}^{t_0+T} \frac{dt}{a(t)}, \qquad R_0 = R(\infty). \tag{1}$$

A null trajectory between the stations uses χ of this distance in either direction. If leg i proceeds at speed $\beta_i c$ relative to local comoving observers, with β_i held constant during that leg, the maximum separation at time T is

$$\chi_m(T) = R(T)\left(\sum_{i=1}^{m}\beta_i^{-1}\right)^{-1}. \tag{2}$$

The $T = \infty$ expression is a supremum. A finite arrival time requires $\chi < \chi_m(\infty)$. For null trajectories this reduces to $\chi_m(\infty) = R_0/m$. A relay cannot improve the bound for an exchange between specified endpoints, since each leg must traverse at least the radial comoving separation of those endpoints.

A proper time τ_j at turnaround j consumes a conformal distance $c\int_{t_j}^{t_j+\tau_j} dt/a(t)$. To first order in small turnaround times the loss in range is the sum of these distances divided by $\sum_i \beta_i^{-1}$, evaluated using the scale factor at each undelayed turnaround. A finite deadline truncates the conformal interval even when all turnarounds are prompt. For all leg counts T is measured

from the first departure to the final arrival on the common FLRW time coordinate.

B. Geodesic first leg

A carrier that maintains constant speed $\beta_0 c$ on the first leg has two-leg range $\beta_0 R_0/(1+\beta_0)$ with a null response. If both carriers maintain $\beta_0 c$, the range is $\beta_0 R_0/2$. These comparisons hold the launch speed fixed.

Alternatively the first carrier can coast on a radial timelike geodesic. Spatial translation symmetry gives $a\gamma\beta = q$, with $q = \beta_0/\sqrt{1-\beta_0^2}$. Thus its speed decays as $cq/\sqrt{a^2+q^2}$, and its range at scale factor a is

$$D_{\rm c}(a) = c\int_1^a \frac{q\,da'}{a'^2 H(a')\sqrt{a'^2+q^2}}. \tag{3}$$

Let $R_{\rm rem}(a) = c\int_a^\infty da'/[a'^2 H(a')]$ be the remaining conformal distance. The limiting first arrival $a_{\rm b}$ for a two-leg exchange with a null response satisfies $D_{\rm c}(a_{\rm b}) = R_{\rm rem}(a_{\rm b})$. The left side increases and the right side decreases. With $m-1$ null legs after the first, the corresponding condition is $(m-1)D_{\rm c}(a_{\rm b}) = R_{\rm rem}(a_{\rm b})$.

C. Fixed receiving band

Consider two null legs, with the return signal emitted at a fixed frequency $\nu_{\rm e}$ at $a_{\rm b}$. The receiving station registers the signal if $\nu_{\rm r} \geq \nu_{\rm min}$, with $K = \nu_{\rm e}/\nu_{\rm min} \geq 1$. Since $\nu_{\rm r}/\nu_{\rm e} = a_{\rm b}/a_{\rm r}$, the latest acceptable reception is at $K a_{\rm b}$. At the limiting separation χ_K the two legs satisfy

$$\begin{aligned} \chi_K &= R_0 - R_{\rm rem}(a_{\rm b}), \\ \chi_K &= R_{\rm rem}(a_{\rm b}) - R_{\rm rem}(K a_{\rm b}). \end{aligned} \tag{4}$$

We assume sufficient received flux for detection inside the band. The model applies to fixed emission and receiver settings. Increasing transmitter power leaves the frequency condition unchanged.

In de Sitter space $R_{\rm rem}(a) = R_0/a$ and Eq. (4) gives $\chi_K/R_0 = (K-1)/(2K-1)$. For the fiducial model below we solve in $x_{\rm b} = 1/a_{\rm b}$. The distance allowed on the return leg is proportional to $\int_{x_{\rm b}/K}^{x_{\rm b}} f(x)\,dx$, with $f(x) = [\Omega_{\rm m} x^3 + 1 - \Omega_{\rm m}]^{-1/2}$. This distance increases with $x_{\rm b}$ since $x f(x)$ increases on $[0,1]$ for $\Omega_{\rm m} < 2/3$. The required return distance decreases with $x_{\rm b}$. There is then one intersection, and a separate deadline T gives the combined boundary $\min[\chi_K, R(T)/2]$.

D. Background model and numerical checks

We adopt $H(a) = H_0[\Omega_{\rm m} a^{-3} + 1 - \Omega_{\rm m}]^{1/2}$ with $H_0 = 67.4\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$ and $\Omega_{\rm m} = 0.315$ [4]. Radiation is neglected. The distance evaluations extend back to $a = 1/4$ and the propagation calculations start at $a = 1$. We obtain the redshift label z_* at each boundary by inverting the relation for light received at t_0 [5],

$$\chi(z) = c\int_0^z \frac{du}{H[(1+u)^{-1}]}. \tag{5}$$

For future integrals we set $x = 1/a$, giving $R_{\rm rem}(a) = (c/H_0)\int_0^{1/a} f(x)\,dx$. Adaptive quadrature and bracketed inversion use a nominal tolerance of 2×10^{-12} in dimensionless variables. Tightening it by a factor of ten in the geodesic calculation changes the quoted radii by less than 4×10^{-11} in relative terms. Direct quadrature in a agrees with the value of R_0 to the same precision. In the de Sitter limit we recover the closed form solution of Eq. (4) and the geodesic distance

$$D_{\rm c}(a) = \frac{c}{H_0 q}\left[\sqrt{1+q^2} - \sqrt{1+q^2/a^2}\right]. \tag{6}$$

III. REACHABLE DOMAINS

A. Null propagation

The present-day fiducial event horizon is 5.114 Gpc. Two exchanges initiated today at different separations are shown in Fig. 1. A target observed at $z = 1$ is at $\chi = 3.401$ Gpc. A null signal sent today arrives after 18.90 Gyr of comoving proper time. The reply remains outside the causal past of station A for all future times.

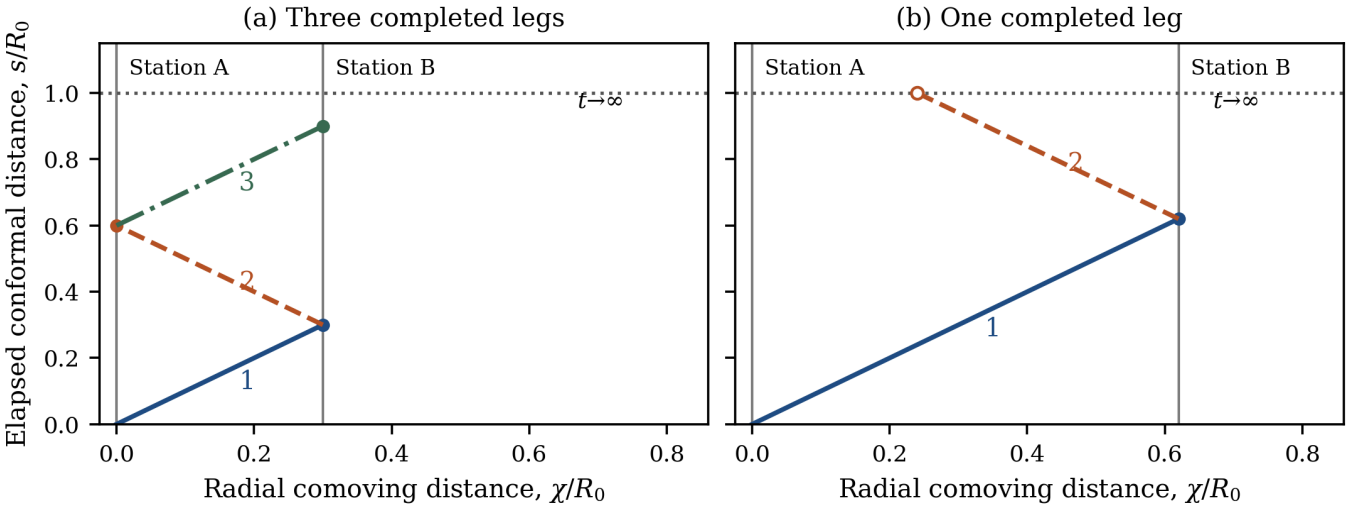


FIG. 1. Null trajectories at two separations. Vertical lines are comoving worldlines. Numbers mark the successive legs, filled points denote arrivals, and the open point is the limiting position of the second signal as $t \to \infty$. The vertical coordinate is accumulated conformal distance since departure.

The uniform comoving volume available to an m-leg null exchange is m^{-3} times the one-leg volume. This ratio is independent of the expansion law as long as there is a finite event horizon, the stations are comoving, and both volumes are inside the sample. As a geometric control, Table I uses a catalog with constant comoving density truncated at $z = 3$, corresponding to 6.506 Gpc. Of its addresses 48.56% are within the one-leg volume. Only 6.07% admit a round trip. An observed catalog would require weights for selection and source evolution.

TABLE I. Limiting comoving distances, present-epoch redshifts and volume fractions for null exchanges with prompt turnarounds. The sample extends to $z = 3$ at constant comoving density.

Number of legs m	χ_m (Gpc)	z_*	Catalog volume (%)
1	5.114	1.869	48.56
2	2.557	0.691	6.07
3	1.705	0.428	1.80
4	1.278	0.311	0.76

Varying Ω_m between 0.308 and 0.322 moves the two-leg boundary from 2.547 to 2.566 Gpc and its redshift from 0.685 to 0.697. Varying H_0 by $0.5\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$ moves the distance by about 0.019 Gpc. The redshift stays fixed at constant Ω_m because the past and future distances in the calculation both scale as H_0^{-1}. The sensitivities vary one parameter at a time by its Planck marginal error.

B. Dependence on transport and deadline

For a maintained launch speed of $0.1c$, changing only the second leg to a null signal increases the two-leg radius by a factor of 1.82, corresponding to a factor of 6.01 in volume Table II. A geodesic first leg and a null second leg reach 0.248 Gpc at the same launch speed. This is within 0.001 Gpc of the geodesic one-leg range. Most of the loss in reach comes from redshifting of the carrier's momentum.

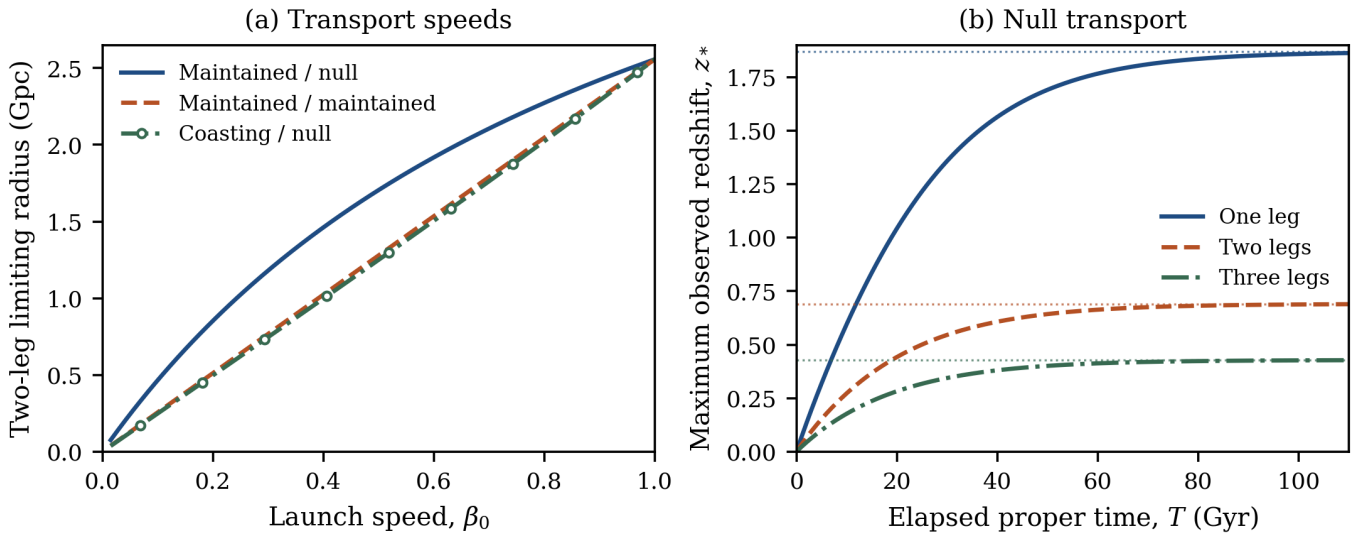


FIG. 2. Left: two-leg radius for the three transport models at fixed initial speed. The second maintained-speed carrier travels at the same local speed as the first. Right: limiting redshifts for null exchanges within elapsed time T. Dotted lines mark the infinite-time limits.

TABLE II. Limiting distances in Gpc for timelike trajectories and mixed null and timelike exchanges. The last row is the null limit of the tabulated cases.

β_0	One leg, maintained	One leg, geodesic	Two maintained	Maintained and null	Geodesic and null
0.10	0.5114	0.2488	0.2557	0.4649	0.2483
0.50	2.5568	1.3332	1.2784	1.7045	1.2496
0.90	4.6023	3.1505	2.3011	2.4223	2.2873
1.00	5.1136	5.1136	2.5568	2.5568	2.5568

The near overlap of two curves in panel (a) of Fig. 2 can be understood from the de Sitter limit. Substitution in Eq. (6) gives $D_c(a) = R_{\rm rem}(a)$ at $a = 2/\beta_0$, so a geodesic carrier with a null response has exactly the same reach as two constant-speed carriers, $\beta_0 R_0/2$. For a positive matter density define $F(x) = R_{\rm rem}(1/x)/R_0$. The integrand $f(x)$ decreases with x, hence F is concave and $F(x) > x$ in the interior. At a fixed remaining fraction y of the light budget the scale factor is larger than in de Sitter space because $x = F^{-1}(y) < y$. A geodesic carrier is therefore slower at fixed y than in de Sitter space at the same launch speed. Its normalized distance is the integral of its speed in units of c from y to 1. The crossing with the straight line y representing the reach of the response consequently lies below $\beta_0/2$ for every $0 < \beta_0 < 1$.

With null transport, a deadline of 10 Gyr permits travel over 44.23% of the infinite-time radius. The two-leg redshift threshold is then 0.272, rising to 0.546 for a deadline of 30 Gyr. The corresponding three-leg threshold at 30 Gyr is 0.344 Fig. 2. Reaching 90% or 99% of the radius takes 40.1 or 80.4 Gyr, respectively, for any fixed number of legs. At large T, $R_0 - R(T) \simeq c/[a(t_0 + T)H_\Lambda]$, with $H_\Lambda = H_0\sqrt{1-\Omega_{\rm m}}$. The waiting time for a target a distance $\delta\chi$ inside the limiting radius thus grows as $-H_\Lambda^{-1}\ln(m\delta\chi/R_0)$ up to an additive constant.

C. Frequency selection

The boundary for a fixed detector band is shown in Fig. 3. With $K = 2$ the fiducial two-leg radius is 1.690 Gpc, giving $z_* = 0.424$. The radius is 66.1% of its unfiltered value and the volume is 28.9%. At $K = 4$ the radius is 2.188 Gpc and the volume fraction is 62.6%. For $K = 2$ the frequency limit is comparable to the infinite-time radius of a third unfiltered leg.

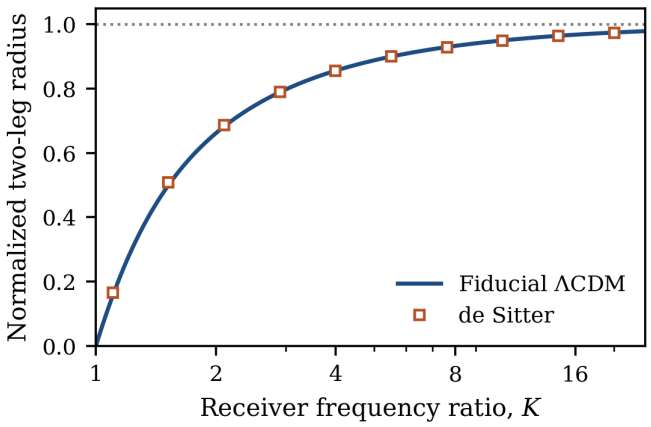


FIG. 3. Two-leg radius with a fixed receiving band on the second leg, normalized to the two-leg null radius without a frequency cut. Squares mark the de Sitter result. The horizontal line gives the unfiltered limit.

IV. APPLICATION TO REFUSED SHIPMENTS

Suppose an addressee refuses an article at receiving office A, which dispatches it back to the return address B at t_0. The sender's signature at B records delivery. Furnishing it to A requires a second transit. Conveying the office's countersignature to B requires a third. Terrestrial mail standards distinguish return of a refused item, obtaining a signature on certain classes of return mail, and furnishing proof of delivery [6, 7]. The countersignature is an additional service condition.

For a refusal processed today, the shell $0.691 < z < 1.869$ admits return of the item but precludes the return of a signature even with ideal null transport. It contains seven eighths of the one-leg comoving volume. In this shell the receiving office can observe the same empty signature field whether or not the sender has taken possession. Recording that field as sender refusal would misclassify every completed delivery in the shell. The station that opened the record can instead select the level of evidence it will require based on the original sender's address. That information is available at the time of refusal.

The band calculation gives a second restriction for an office that uses fixed-frequency electronic proof of delivery. At $z = 0.5$ the office can receive a null signature in 24.97 Gyr, within a 30 Gyr retention period. However, the received frequency is lower than the emitted frequency by a factor of 2.641. For a receiver with a cutoff at half the emission frequency, the signal arrives below the cutoff. Either a wider band or a change in emission frequency can make this example eligible. No such changes make an unfiltered signature from $z = 1$ arrive.

A 30 day hold for obtaining a signature changes the ideal two-leg distance by about 0.006 pc. At the boundary the turnaround is at $a = 2.09$, so the conformal time consumed by a local handling interval is less than today. The frequency cut changes the distance by 0.867 Gpc at $K = 2$. For the modeled service, the acceptance range is sensitive to the choice of receipt channel even when the handling interval is immaterial.

V. DISCUSSION

The cut at finite distance depends on the continuation of accelerated expansion. As a comparison, suppose the expansion history is identical to the fiducial model through $a = 2$, and the energy in the vacuum component subsequently dilutes as pressureless matter. The density is continuous across the transition. In the matter-dominated future $H(a) \propto a^{-3/2}$, so the future null distance diverges. Any finite null exchange can then be completed given enough time, despite identical distances on today's past light cone. The reach of a coasting carrier remains finite in the same model because its speed supplies another factor of a^{-1} to the range integrand.

We neglect peculiar motion, lensing, signal attenuation and the lifetime of stations. A relay can establish custody at its own location. Proof of receipt at the original address requires a signal from that address. The terrestrial provision to keep the return-receipt card attached to certain undeliverable articles puts both records at the original address. In the one-leg shell, the refusing office can close a delivered case only if its rules accept a receipt held by the sender.

DATA AVAILABILITY

The results use deterministic quadrature. The background parameters, propagation law and sample selection are specified in Sec. II. The volume fractions are obtained by integrating a constant comoving density.

[1] T. M. Davis and C. H. Lineweaver, Publ. Astron. Soc. Aust. **21**, 97 (2004).
[2] L. M. Krauss and G. D. Starkman, Astrophys. J. **531**, 22 (2000).
[3] J. Y. Halpern and Y. Moses, J. ACM **37**, 549 (1990).
[4] Planck Collaboration, Astron. Astrophys. **641**, A6 (2020).
[5] D. W. Hogg, *Distance Measures in Cosmology*, arXiv:astro-ph/9905116 (1999).
[6] United States Postal Service, *Domestic Mail Manual, Sect. 507.1.8: Returning Mail*, Postal Explorer (2026a).
[7] United States Postal Service, *Domestic Mail Manual, Sect. 503.6: Return Receipt*, Postal Explorer (2026b).